

Legal Restrictions and Optimal Contracts*

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Preliminary.
Comments are very welcome!

Abstract

Real-world transactions are often limited: Not all contracts are legal. As a result, I show economic examples where the only way to achieve first-best is as follows. First, the parties contractually commit to an inefficient different outcome. Then, they secretly violate the contract and coordinate play on first-best. If the parties were to contractually commit to first best, they would fail to achieve it. So, first best can only be achieved by signing a contract that appears to be over-specified, as it contains unenforceable prescriptions. I develop a novel theory of how contracts are shaped by legal constraints, and derive conditions such that the phenomenon I identify cannot take place. For example, this is the case if the court's verification of events is costless and if all contracts that can be stipulated are legal. Under this condition, the standard contract-theoretical assumption that parties stipulate only enforceable contracts is without loss of generality.

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1 Introduction

Suppose that, before taking part to an economic interaction, economic agents may sign a contract to Pareto-improve upon the equilibria of their game. In the real world of contracts, not all possible contractual stipulations are legal, so that contractual transfers need not always be enforceable. Because of this, my paper describes simple economic settings, where the only manner for the involved parties to achieve the first-best is to adopt an entirely novel and completely non-obvious contractual mechanism. The parties stipulate a contract that explicitly *prohibits the first-best*, but secretly agree to *breach the agreement in equilibrium* so as to achieve the optimal outcome. Most importantly, the optimal outcome can *only* be achieved with these contracts that are violated in equilibrium. In my economic settings, the contract prescribing that the parties coordinate on the first-best outcome *fails to induce the first-best*.

These findings are surprising, and contradict the natural supposition that it would be a waste of resources to stipulate unverifiable or unenforceable contractual clauses, the optimal outcome can be achieved with these contracts that are violated in equilibrium. In my economic examples, optimal contracts are violated in equilibrium, and hence necessarily contain unverifiable and unenforceable prescriptions.¹ My results show that the usual contract-theoretical definition of contracts as transfer schemes that depend on signals verifiable in court is too restrictive, because it is equivalent to assuming that the players may only sign contracts that do not include unverifiable prescriptions. Most importantly, my results show that the issue of contract enforcement cannot be settled by considering verifiability constraints only. The constrain that not all possible contractual stipulations are legal is potentially a more important limitation for contract enforcement. This paper introduces a general contract-theoretical representation that captures all relevant enforceability constraints, both technological ones (such as verifiability constraints), and institutional ones

¹This fact is also of interest for the foundations of contract incompleteness, because it shows that the players should not necessarily sign an incomplete contract when complete contracts are unenforceable. Following seminal work by Williamson (1985), Grossman and Hart (1986), and Hart and Moore (1990), the literature on contract incompleteness has grown vast (see, for example, the survey by Tirole, 1999). Transaction and complexity costs have been studied by Anderlini and Felli (2001), among others.

(such as legal constraints).

Usual contract-theoretical models define contracts as (contingent) transfer schemes that depend on signals verifiable in court. Here, contracts are defined as transfer schemes that depend on the players' choices in the game (and possibly also on signals verifiable in court). The main methodological innovation of this paper is the introduction of an 'enforcement operator' F that maps each one of the contingent transfers stipulated in the agreed contract into actually enforced transfers. Hence, I do not assume that parties only sign enforceable contracts, but I let my model specify what would happen in court when the sign contract includes unenforceable contingent-transfers, possibly distinguishing across different contingencies.² In this paper, I use this novel framework to ask: When is the assumption that parties sign only enforceable contracts without loss?

Dubbing 'enforceability principle' the object of this enquiry, Theorems 1 and 2 identify mathematical properties of the enforcement operator F that determine whether this principle holds or not.³ I show that it does, if all contracts are legal (or if the only legal limitations to enforcement is that parties liability is bounded), under standard assumptions about verifiability. Specifically, suppose that one models the verifiability constraint as an information correspondence P that describes what a Court verifies about the players' choices (i.e., contingencies). For every realized contingency, this correspondence identifies the set of contingencies that a Court cannot exclude have occurred. The Court can then verify which transfers should be enforced only if the agreed contingent transfers are the same across the contingencies she deems possible.⁴ The usual assumption in information economics is that information correspondences are partitional. If the Court's information satisfies this condition, Proposition 1 shows that the enforceability principle holds if all contracts are legal, or if the only legal limitations to enforcement is that parties liability

²This methodological innovation is novel and distinct, but shares the same broad motivation with the research question of Anderlini, Felli and Postlewaite (2007, 2011). They ask whether Courts should always enforce transfers stipulated in contracts, and determine the optimal rules of contract enforcement in specific contractual settings. Here, instead, Courts are constrained by general legal liability constraints, and cannot finesse enforcement rules to the equilibria of the specific game played by the contractual parties.

³Specifically, Theorem 2 shows that the enforceability principle holds if F is idempotent, so that $F \circ F \circ t = F \circ t$ for all contracts t .

⁴I.e., in the standard language of information economics, it is needed that the transfer scheme is measurable with respect to the Court's information structure.

is bounded. Hence, within this standard domain, usual contract-theoretical modelling is without loss.

Information correspondences are usually assumed partitional when standard game-theoretical players are concerned. Courts however are not standard game players: For example, they are bound by codified rules of evidence, and can only produce decisions based on hard evidence, and not on personal opinions, or equilibrium beliefs. So, it may be reasonable to allow for the possibility that Court's information correspondences are not partitional, as was earlier suggested by Geanakoplos (1989) and Shin (1993), and will be later explained in details. Proposition 2 shows that, when all contracts are legal, the enforceability principle holds even if the correspondence P is not partitional, as long as it is transitive, i.e., as long as $s'' \in P(s')$ and $s' \in P(s)$ implies that $s'' \in P(s)$ for any triple s, s', s'' of players' choices. In words, when P is transitive, it must be the case that, whenever any outcome s realizes, the Court knows what she would know had any other outcomes s' realized. It is not obvious whether this 'axiom of positive introspection' is reasonable when describing a Court's information correspondence. Example 3 argues that this axiom may fail when the Court's evidence gathering and processing activities are costly. Intuitively this is because verifying an event may be less costly than verifying that the event can be verified, as this introduces one additional layer of costly verification.

Further, and most importantly, this paper identifies legal rules that can be related to the mathematical characteristics of the enforcement operator that induce a failure of the enforceability principle, even in the standard case that the Court's information correspondence is partitional. Specifically, this paper's model is used in section 5 to generally describe the effect of two important legal principles that may sometimes limit contractual enforcement. The first one, dubbed here 'individual liability,' is the principle that a single individual may not be sanctioned for a collective breach of contract, unless it is verified

that she is among those who violated her contractual commitment.^{5,6} The second principle, dubbed here ‘damage compensation’, prescribes that Courts do not enforce contractual transfers (liquidated damages clauses) that are punitive in nature. So, even if it is verified that one breached contract, she cannot be sanctioned in excess of verified harm or of the foregone profits suffered by her counterparts because of her breach of contract.^{7,8}

These two legal principles make some contractual transfers illegal or unenforceable. As I show in the next section, each one of these principles may generate economic examples in which the enforceability principle fails, in the sense I described above. Before presenting these examples in details, it may be important to pause and underline the novelty with respect to the existing contract-theoretic literature. First, unlike previous literature, these examples are not based on (nor they include) contract renegotiation. The contract violation that leads the parties to achieve first best is secret, here, and no renegotiation ever takes place. Second, and in stark opposition to the large literature on contract incompleteness, contracts are not incomplete, here. To the contrary, contracts are “overspecified” as they

⁵Contractual liability normally follows from individual contractual commitments, so that individual liability may be considered the default rule. An exception is outlined in the regulation of ‘negotiable instruments’ (i.e. unconditional promises or orders to pay a fixed amount of money). The Universal Commercial Code 3-116 states that “Except as otherwise provided in the instrument, two or more persons who have the same liability on an instrument [...] are jointly and severally liable [...]” While not predominant in contract law, joint and several liability may apply in tort law, e.g. the Comprehensive Environmental Response Compensation and Liability Act.

⁶The question of which liability rule is economically efficient in different environments has received significant attention in law and economics. For example, Kornhauser and Revesz (1989) show that a variety of rules are optimal when the court is fully informed, whereas Emons and Sobel (1991) determine rules that decentralize first best. Besides being detrimental to defendants, Kornhauser and Revesz (1994) show that the joint and several liability rule may also be detrimental to plaintiffs because it may stifle or complicate out-of-court settlement.

⁷For references to this, see the Uniform Commercial Code (sections 2-718 and 2A-504), the Restatement (Second) of Contracts section 356, and 347 cmt. a: “Contract damages ... are intended to give [the plaintiff] the benefit of his bargain by awarding him a sum of money that will, to the extent possible, put him in as good a position as he would have been in had the contract been performed.” In virtually all legal systems outside the United States, punitive damages are either excluded or play a very minor role. Even in the US, punitive damages are normally restricted to cases of reckless conduct, e.g. drunken driving.

⁸In the economics literature on damage compensation, Aghion and Bolton (1987) demonstrate the role for stipulated damage provisions in excluding competitors. Stole (1992) provides a signalling explanation for the court enforcement of liquidated damage terms (as long as they do not significantly exceed actual losses). Lewis and Sappington (1999) show that lender’s deep pockets mitigate judgment-proof problems arising with budget-constrained producers.

include clauses that cannot be enforced.^{9,10} Third, and again in stark opposition with the existing contract-theoretical literature. The economic examples presented here are not based on relational contracts “completing” possibly incomplete contracts. Here, contract are literally breached on the equilibrium path; and the examples presented here are one-shot interactions, where no relational or reputational considerations can play any role.¹¹

On the other hand, the failure of the enforceability principle described here is mathematically and conceptually related to known violations of the revelation principle. Specifically, Green and Laffont (1986) study an environment where a single informed agent communicates to a principal who commits to a decision rule that specifies a choice for any received message. But unlike in the standard set up, e.g., Myerson (1979), the agent’s message is not cheap talk. It is constrained by correspondence, describing, for each state of the world, the set of states to which the message may belong. If the correspondence is such that the agent’s message is unconstrained, the revelation principle holds, and it is without loss to consider only truthful (direct) mechanisms. Green and Laffont (1986) show that the key axiom is transitivity: the revelation principle holds in this environment if and only if the correspondence is transitive. Likewise, this paper shows that, if all contract are legal, the revelation principle holds if the Court’s information correspondence is transitive.

2 Economic Examples

The first example illustrates the possibility that players circumvent legal restrictions by signing contracts that they plan to secretly violate in equilibrium, to achieve first best. The specific legal principle I consider here, named ‘individual liability’, limits the sanctioning

⁹As later explained, these unenforceable clauses can be related with small-print, ‘boiler-plate’ clauses that are usually included in contracts despite being often ignored by the contracting parties.

¹⁰While this paper determines conditions under which optimal contracts include unverifiable prescriptions, Bernheim and Winston (1998) follow the ‘opposite route’ of identifying games where the optimal contracts exclude verifiable prescriptions. According to a commonly held view, imperfect verifiability could make complete contracts ineffective, because it constrains enforceability. Bernheim and Winston (1998) identify a further (purely strategic) source of contract incompleteness whereby the players may optimally choose contracts that do not restrict their choice to the maximal extent allowed by verifiability constraints.

¹¹Following the seminal insights by Bull (1987), and the formalizations by Baker, Gibbons and Murphy (2002) and Levin (2003), the literature on relational contracts is steadily growing. See Halac (2012) for one of the latest papers in the area.

	H	L
H		
L		

Figure 1: Simple Team Problem

of each single individual for a collective breach of contract, unless it is verified that she is among those who violated her contractual commitment.

Example 1 (Individual Liability) I begin the exposition by laying down a simple multi-agency problem. Each of two agents makes a private unverifiable investment that determines the quality of jointly produced output sold to a buyer, who may choose to buy (action B) or not (action N). For instance, each agent may be a producer of intermediate products necessary in the production process of the buyer; or each agent may be a professional, and the agents' services are complementary for the buyer. Joint output is verifiable in court, and it is high if and only if both agents' investment is high (action H). In the socially-optimal outcome, the two agents play H and the buyer buys the output. However each of the two agents has a private incentive to underinvest (action L , for low investment), and the court cannot distinguish private investments. The verifiability structure associated with the agents' choices is described in Figure 1.

If agents are only individually liable, joint commitments are not legally enforceable, and it will not be possible to effectively deter underinvestment. Consider, in fact, the contract between agent 1 and the buyer, and suppose that agent 1 commits to play H , but then breaches the agreement and plays L . If taken to court, agent 1 can defend herself by blaming agent 2 for the low output. Since investments cannot be identified, there is no way to show that agent 1 violated her contract. Note that in this multi-agency problem, individual liability requires that the principal signs bilateral contracts with each agent separately, as in Prat and Rustichini (2003).

We now elaborate this simple model, and suppose that each agent can costly monitor the other agent's investment (action M), or choose to overlook her conduct (O). Besides productive effort, also the choice of monitoring is a private action, so that it cannot be

	HM	HO	LO	LM
HM				
HO				
LO				
LM				

Figure 2: Team Problem with Monitoring

directly verified. Despite this, if agent i monitors agent j , then she is able to gather enough hard evidence to verify that agent j has shirked, whenever this is the case. Specifically, we introduce a binary verifiable signal x_i for each agent i , and say that $x_i = l$ if and only if agent i plays L and agent j plays M , otherwise $x_i = 0$. This is appropriate, for instance, to represent the choice by agent j to gather all available evidence that i shirked and collect it in a report, without including any evidence that i worked hard. If j 's report fails to show that i shirked, it can be either because i worked hard or because j did not spend enough effort in gathering evidence.¹²

The verifiability structure associated with the agents' choices is described in Figure 2, where, for instance, the upper-left box gathers all action profiles inducing high output, and the step-like set includes all action profiles where output is low, but $x_1 = x_2 = 0$, so that neither of the agents can be blamed for this. The payoffs are as follows. Each agent pays a cost c when playing H , and suffers a loss l if playing M . If the output is of high quality, it is worth y to the buyer, otherwise it is worth 0. In either case, it is worth 0 to the agents. If the transaction is concluded, the buyer receives the output and pays p to each agent.¹³ To make the problem meaningful, we assume that $p > c$, and that $y > 2p$, otherwise it would not be efficient that agents exert high effort.¹⁴

The first-best outcome is (HO, HO, B) : the two agents invest at the cooperative levels,

¹²We discuss later the alternative possibility that agents' monitoring efforts are directly verifiable.

¹³As is often the case in these settings, we implicitly assume that the quality of final output is not contractible in the sale contract; possibly because of unforeseen contingencies.

¹⁴Besides making trade beneficial to all parties, this implies that the damage incurred by the buyer if buying low-quality output is larger than the joint gains of the agents for playing L . Hence this example is not affected when contractual transfers are restricted to damage compensation.

without wasting resources monitoring each other, and the counterpart buys the output. But as in the simpler multi-agency problem, the contract prescribing cooperative investments without monitoring, will fail to deter underinvestment. Again, if an agent breaches the agreement and underinvests, there is no way to blame her for low output.

The players can achieve the first-best outcome (HO, HO, B) only by signing the contract that prescribes cooperative investment levels, and that requires the agents to monitor each other. In equilibrium, each agent will secretly violate the contract, and play HO , so as to cooperate without wasting her effort on monitoring. Knowing this, the buyer will play B , and the first-best will be achieved. The agents' contract violation cannot be verified in court: since output is high, no agent can be blamed for low output, and hence the absence of monitoring efforts cannot be verified.

Moreover in equilibrium this apparently contradictory mechanism is effective in deterring underinvestment. Suppose that the buyer plays B , agent 2 plays HO , and agent 1 deviates from the candidate equilibrium to play LO . When taken to court by the buyer, agent 1 will not be able to discharge herself of the accusation of breaching the contract. Specifically, agent 1 will be unable to show in court that agent 2 has underinvested. Hence the court will conclude that either she did not gather enough evidence on agent 2's conduct, or that she is really the one who underinvested. In either case agent 1 is liable for breaching her contractual commitment which requires her both to make the high investment, and to monitor agent 2.^{15,16} ◇

In the above multi-agency problem, the agents will not be able to achieve the first-best if the monitoring technology is such that the Court can directly verify whether a player has monitored the opponent or not. When this is the case, in fact, if an agent does not monitor

¹⁵One interesting feature of this equilibrium is that, although each agent is only individually liable, if one agent underinvests, then both agents are penalized. This occurs because the buyer can independently verify that *each of the agents* has breached contract, either because she has underinvested, or because she has not monitored the other agent. Under joint liability, it would be possible to punish both agents if the buyer could show that *at least one of them* has breached contract.

¹⁶Peer monitoring has been suggested as a possible means to alleviate moral-hazard (see for instance Laffont and N'Guessan, 2000, and Rahman, 2012), but if agents monitor each other in equilibrium, the first-best cannot be achieved. Our analysis identifies a setting where there is no need for actual monitoring in equilibrium, so that the first-best is achieved.

the other agent, the buyer will directly verify in court that the agent did not monitor, and will collect the fine. As a result, only a second-best outcome can be achieved: both players commit to work hard and monitor each other and, in equilibrium, they fulfill their contractual obligations, to avoid being penalized. The same reasoning shows that the first-best cannot be achieved with any contract by which agents commit to a verifiable action unrelated with output. The inclusion of monitoring in the contract does not only serve the role of allowing the buyer to collect fines even though effort is not verifiable, it is crucial that the contract allows collection if and only if output is low.

While setting the above example within detailed empirical analysis is out of scope of this paper, it is not difficult to think about anecdotes in which agents are asked to monitor each other to provide incentives that they all exert effort cooperatively, and to avoid that they start blaming each other if something goes wrong, so that identifying who precisely should be sanctioned becomes very difficult and costly. A simple example can be taken from university life. One of the aims of the students' code of honor is to prevent students from cheating and earning an undeserved high grade. However, it is usually difficult to directly verify whether a student cheats, and often the only instance in which cheating can be verified is when two or more students turn in identical exam answers. But in this event, it is impossible to establish which student copied from which. The first-best can still be achieved by overspecifying the code of honor (i.e. the "contract") by requiring students not only not to cheat, but also to take steps to prevent anyone else from cheating from their work. Students are penalized for failing to take such steps only if there is evidence that cheating has occurred and, in practice, no such steps are ever actually taken.¹⁷

The previous example has considered a simple legal environment, in which contracts

¹⁷A different economic scenario for which Example 1 may be of relevance is the following one. A private owner renovating real estate will typically not hire independent contractors for the different tasks involved in the renovation (planning, building, electrics, gas and water, ...), because it would be a nightmare to establish who is to blame if something went wrong. Usually, a building company is hired, which then subcontracts the renovation to separate individual professionals. Within the subcontracting agreement, it is usually understood that the subcontractors check that each other work's is correctly undertaken. No direct supervision by the subcontracting company is in place, and it is reasonable to presume that subcontractors do not waste too much effort in monitoring each other either. While this interaction is of more difficult interpretation as subcontracting is often a long term relationship, the formulation of Example 1 above may still be of some relevance.

establishing joint liability by the agents are illegal. To circumvent this legal constraint, the agents sign an overspecified contract, which they violate in equilibrium to achieve first best. The aim of the contract is to reconstruct the incentives to make the agents responsible for each other actions in the productive task. We now show a different example which illustrates a similar phenomenon, when contracts are constrained by a damage compensation rule. According to these rules, transfers are limited to compensate a player for the damages she suffered because of breach of contract. Punitive transfers are ruled out. The example consists of a simple hold up problem. When the damages incurred because of the failed investment cannot be precisely verified, the players may be willing to sign an overspecified contract, in which they contract an amount of investment which is excessive for the first best. Then, they secretly deviate so that the agent makes first best investment, and the damages relative to the overspecified investment cannot be verified. Nevertheless, further deviation is penalized, because the additional damages would be verified, and the repayment of these additional damages is sufficient to deter the agent from deviating from first best.

Example 2 (Damage Compensations) Consider the following simple hold up problem. Player 1, the agent or seller, produces a good for player 2, the principal, or buyer. The quality x of the good may have one of three values, 0, 1 or 2, and it is not verifiable. Player 1 may or may not exert effort in two different activities, A and B . Exerting effort in each of the two activities is costly to player 1, and contributes a value of one to the quality of the good, so that $x = e_A + e_B$, where $e_j = 0, 1$ for $j = A, B$. While effort in task A is verifiable in court, effort in task B is not. For instance, the activity A may require the use of a simpler (and hence contractible) technology than activity B , or it may consist in a technology upgrade, or in a capacity increment; whereas activity B may require some expertise than it is more difficult to describe in court.

For brevity, we do not make the compensation of player 1 explicit, we assume that producing a good of higher quality induces gains from trade; i.e., $v(2) - c(1) - c(1) \geq v(1) - c(1) \geq v(0)$. It is immediate that the efficient outcome $x = e_A + e_B = 2$ cannot ever be achieved, as the effort level e_B and the quality x are not verifiable. We shall henceforth focus on the implementation of the outcome $x = e_A + e_B = 1$. If transfers are unrestricted,

this outcome can plainly be achieved by a contract in which player 1 commits to make a sufficiently large penalty transfer to player 2 if failing to exert the verifiable effort $e_A = 1$.

In line with the motivation for this example, suppose however that transfers are not implemented by the court, when they exceed the compensation of verified damages. Then, evidently, the outcome $x = e_A + e_B = 1$ cannot be achieved by any contract by which player 1 commits to exert only the verifiable effort $e_A = 1$ (or, a fortiori, only the unverifiable effort $e_B = 1$). Suppose that player 1 violates the agreement and plays $e_A = 0$. Although, this can be verified in court, player 2 cannot verify that she suffered a damage. In fact, quality x is not verifiable, and player 2 cannot verify e_B either. So, it cannot conclude that the good's quality is zero, and that she suffered a loss. There are no verified damages, and hence the court does not implement any transfer from player 1 to player 2.

On the other hand, the efficient outcome can be achieved with the contract by which player 1 commits to pay the transfer $v(2) - v(1)$ to player 2 if failing to exert both the verifiable effort $e_A = 1$ and the unverifiable effort $e_B = 1$. In equilibrium, player 1 violates the contract by only supplying the efforts $e_A = 1$ and $e_B = 0$. In fact, e_B cannot be verified and hence, player 1 cannot be taken to court for violating the contract. But if player 1 undersupplied also effort e_A , then player 2 would be able to take player 1 to court and verify the following. If player 1 had exerted both efforts $e_A = 1$ and $e_B = 1$, the quality of the good would be $v(2)$. However, it can be verified that player 1 exerted low effort in activity A . Hence, it is verified that quality is (at most) $x = 1$, and that player 2 suffered a damage $v(2) - v(1)$. Thus, the court implements the transfer $v(2) - v(1)$, which is a sufficient deterrent for player 1 to exert effort $e_A = 1$, as $v(2) - c(1) - c(1) \geq v(1) - c(1)$ implies that $v(2) - v(1) \geq c(1)$.

As in Example 1, we have shown that the only way for the players to achieve a desirable outcome ($x = e_A + e_B = 1$), is to contractually commit to a different outcome ($x = e_A + e_B = 2$), to then secretly violate their contractual commitment in equilibrium. If the players contractually committed to the desired outcome, then they would fail to achieve it. ◇

The above simple example resonates with the significant evidence that terms of trade in commercial contracts are often overspecified. Many boiler-plate, small-print clauses are usually ignored by trading partners, but they are often included by their lawyers in the sales contract to overcome verifiability issues and simplify establishing the amount of damages suffered in case of a more significant breach of contract.¹⁸ (For a study of the divergence between law and sales practices, see Keating, 1997). While there is a significant legal and economics literature that studies the consequences of the unenforceability of punitive transfers,¹⁹ the observation that contracts may be overspecified to overcome these enforceability issues has never been picked up by contract theorists.

We have here seen two examples in which some contracts are illegal, and the players need to circumvent this restriction in order to achieve first best. Specifically, we have seen that they sign overspecified contracts and then violate them in equilibrium to achieve first best. The next section will argue that the fact that some contracts are illegal is a necessary feature of these examples. When all contracts are legal, then the enforceability principle we have discussed in the introduction holds (at least, as long as the court's verification of events does not entail too significant a cost, as we shall explain later). Players can achieve any outcome by means of contracts whose prescriptions are enforced by the court, at least, on path. I also provide a converse result, characterizing under which conditions, examples such as 1 and 2 can be constructed, in which the only means to achieve a first best outcome is for the players to contractually commit to a different outcome to then secretly violate the contract in equilibrium.

¹⁸In fact, these breaches of contracts are usually settled out of court, so that the boiler-plate clauses are used mostly to make the threat of going to court credible in the out-of-court bargaining process.

¹⁹See, for example, the symposium on 'Economic Loss' published on the *International Review of Law and Economics*, 2007, vol. 27.

3 The Model

Consider a strategic form game $G = (I, S, u)$, where I is the set of players, S is the (finite) strategy space, and u are the utility functions.²⁰ In anticipation of playing game G , the players may choose to stipulate a contract in order to Pareto improve upon the Nash Equilibria of G . A contract is a transfer scheme $t : S \rightarrow \mathbb{R}_+^{I \times I}$, where $t_{ij}(s)$ denotes the commitment of player i to make the transfer $t_{ij}(s)$ to player j if the strategy profile s is realized. From the transfer scheme t , we call $n_{ij}(s) = t_{ij}(s) - t_{ji}(s)$ the pairwise ‘net transfers’ from i to j , so that, clearly $n_{ij}(s) = -n_{ji}(s)$.²¹ The net aggregate transfers $n|t : S \rightarrow \mathbb{R}^I$, can be immediately derived: for all players i , $n_i(s)|t = \sum_{j \in I} t_{ij}^j(s) - \sum_{j \in I} t_{ji}^i(s)$. Note that, by construction, the contract is *budget-balanced*: $\sum_{i \in I} n_i(s)|t = 0$ for all s . This assumption is without loss of generality, because the game can always be expanded to include a player who does not make payoff-relevant actions, and whose only role in the game is to break the budget constraint.²² Were the contract t enforced, the players’ utilities when choosing their strategies would be $u + n|t$, and we introduce the notation $G|t = (I, S, u + n)$.

An implicit assumption in contract theory is that parties restrict attention to enforceable contracts when trying to improve upon the equilibria of the game G . For example, when all contractual transfers are legal, a contract is enforceable as long as a Court can verify what transfers the parties intended that be implemented. Verifiability constraints are usually modeled by introducing a set of verifiable signal realizations X , where the realization x depends on the strategy profile played s . Then, modelers restrict attention to contracts defined as transfers $t : X \rightarrow \mathbb{R}^{I \times I}$, with the idea that courts cannot enforce transfers that depend on facts that they do not observe. The contract space is then sometimes further

²⁰To the extent that the underlying game G may be expanded so as to include the possibility of sending messages, our model subsumes the message game approach adopted, for instance, by Green and Laffont (1977). The restriction to games of complete information is made for expositional ease only, it is immediate to expand our model to allow for general Bayesian games.

²¹We distinguish transfers from player i to player j , instead of just considering net transfers to players, because we want to cover the case of individual liability (Example 1), in which the identity of the player making a transfer is crucial to assess whether the contractual prescription is legal or not.

²²In Holmstrom (1982) for instance, the members of a team achieve the cooperative outcome by appointing an agent, external to a team, with the right of collecting fines from the members of the team if any of them has failed to cooperate.

restricted to account for the fact that not all contractual transfers are legal. (For example, one can bound contractual transfers, in recognition that parties cannot be expropriated of their fundamental rights).

We call the implicit assumption of restricting the contract space to only contracts that can be enforced, the *enforceability principle*. The previous section presented two economic examples in which this principle is violated: Optimality can be achieved only with contracts that are not enforceable, and this failure of enforceability takes place on the equilibrium path. The analysis of the next section will identify general environments in which the enforceable principle may or may not be violated. In order to formally define this principle, I now need to introduce the distinction between the stipulated transfers and *enforced* transfers. I see this innovation as the most important methodological innovation of this paper.

I say that a legal rule \mathcal{L} maps every game $G = (I, S, u)$ into an transformation operator F . The operator F determines how stipulated contracts are enforced by Courts. Hence, F maps every stipulated contract t (defined on S , as above) into an enforced contract $F[t]$. As above, we introduce the net aggregate transfers $n|F[t]$ such that $n_i(s)|F[t] = \sum_{j \in I} F[t]_i^j(s) - \sum_{j \in I} F[t]_j^i(s)$, so that when stipulating contract t in game G under legal rule \mathcal{L} , the players utilities when choosing their strategies are $u + n|F[t]$. (As well as the notation $F[t]$, I also adopt the notation $F \circ t$.) I assume that $F \circ t^0 = t^0$, where t^0 is the null-contract, for which $t_{ij}^0 = 0$ for all i, j . But I do not place any further restrictions on F .²³

It is important to underline the difference between my approach and the standard approach of ruling out unenforceable contracts. Here, I allow that unenforceable contracts be signed by the parties, but I explicitly determine which contractual transfers the court implements, and which one it does not. The difference is crucial as none of the contracts employed in my leading examples is enforceable, and yet they are unique the contracts that

²³We also note that, for some applications such as Example 2, the operator F is also a function of the game G . In that example, in fact, transfers are constrained to liquidate verified damages, and hence whether a transfer is implemented by the court or not depends on the game G played, through its payoff function.

achieve first best.

In order to see how my construction can be used, I now relate to ‘standard’ contract theory. When all contracts are legal, a simple way to model verifiability constraints is by means of an information correspondence $P : S \rightarrow 2^S$; if the players play profile s , then the court can only conclude that no profile $s' \notin P(s)$ has been played, but cannot distinguish among the strategy profiles contained in $P(s)$.²⁴ Here, a contract t is verifiable (and enforceable) if t is measurable with respect to P : When $s' \in P(s)$, it must be that $t(s') = t(s)$. In other terms, the contractual transfer t are only a function of the range of P , which can be equated with the set of signals X .²⁵

It is easy to represent verifiability within my construction. For any game G , given the Court’s information P , and the stipulated contract t , every transfer $t_{ij}(s)$ is enforced, i.e., $F[t]_{ij}(s) = t_{ij}(s)$, if and only if $t_{ij}(s) = t_{ij}(s')$ for all $s' \in P(s)$. When, on the basis of its information, the Court can conclude the contingency in which i committed to pay $t_{ij}(s)$ to j has realized, the Court enforces the transfer $t_{ij}(s)$, so that $F[t]_{ij}(s) = t_{ij}(s)$. There are many reasonable assumptions on how to complete $F[t]_{ij}(s)$ for contracts t and outcomes s for which the Court cannot determine which commitment i has made to j in the realized contingency. For brevity, I say here that $F[t]_{ij}(s) = 0$ when there is $s' \in P(s)$ such that $t_{ij}(s) \neq t_{ij}(s')$, so that the court simply voids transfers that it cannot verify. I will revisit this matter later in this section. Notably, verifiability is the minimum requirement for contract enforcement. Absent any legal constraints, F is such that $F[t]_{ij}(s) = t_{ij}(s)$,

²⁴This approach has been used for example by Bernheim and Whinston (1998), who only considered truthful and partitional verifiability structures, i.e. correspondences P such that $s \in P(s)$ for any outcome s , and such that $P(s) \cap P(s') \neq \emptyset$ implies $P(s) = P(s')$. Here, I will not assume that P are always partitional, but I will maintain that they are always truthful.

²⁵This formulation implicitly assumes that the relationship between signal realizations X and strategy profiles S is deterministic. It is conceptually easy to expand my model to allow also for stochastic signals verifiable by the Court. It suffices to include an additional player, nature, who moves after the players I played, and whose stochastic move is described by a probability distribution $p(x|s)$ over the the set X , given the strategy profiles S played. Then, contracts can be defined as transfer schemes $t : S \times X \rightarrow \mathbb{R}_+^{I \times I}$, a suitable information correspondence $P : S \times X \rightarrow 2^{S \times X}$ describes verifiability constraints (in the usual case, in which signal realizations are verifiable and strategy profiles are not, $P(s, x) = S \times \{x\}$ for all s and x), and players choose their strategies according to expected utilities $u(s) + E[n(s, x)|F[t]]$, where the expectation is taken with respect to the chance of nature $p(x|s)$. However, studying the intricacies of this generalized model would confuse the main points made in this paper. So, while noting that this model may be quite valuable as it also covers Bayesian games, I postpone its study to future research.

if and only if $t_{ij}(s) = t_{ij}(s')$ for all $s' \in P(s)$. In the presence of legal constraints, it may be that some verifiable transfers $t_{ij}(s)$ are not enforced by the court, and hence that $F[t]_{ij}(s) = 0$ despite $t_{ij}(s) = t_{ij}(s')$ for all $s' \in P(s)$; but the opposite cannot ever happen.

I now introduce my definitions of achievable and enforceable contracts. An outcome s can be achieved if it is a Nash equilibrium of the game G modified by a contract t , which may or may not be enforceable under the verifiability and legality constraints represented by the operator F . A contract t is enforceable if all its stipulated transfers are enforced by a Court, despite the constraints imposed by legality and verifiability.

Definition 1 *Given the game G and the operator F , a strategy profile s is achievable with a contract t , if it is a Nash equilibrium of the game $G|F \circ t = (I, A, u + n|F \circ t)$.*

Given the game G and the operator F , contract t is enforceable at s if $t(s) = F \circ t(s)$. The contract t is (globally) enforceable if $t(s) = F \circ t(s)$ for all $s \in s$.

The ‘enforceability principle’, implicitly assumed in contract theory, presumes that given a game G and operator F , the players can restrict attention without loss to enforceable contracts, when determining whether an outcome can be achieved by means of a contract. The enforceability principle can then be formalized as follows, here.

Enforceability Principle *Given game G , and operator F , any achievable outcome s can also be achieved by an enforceable contract t .*

Our examples 1 and 2 show environments where a (socially-optimal) outcome s can only be achieved by a contract t that is not enforceable at s . In fact, in order to achieve the first best s , the parties need to sign a contract that prescribes some transfers if s is played, and yet in equilibrium they play s and these transfers are not established. The next section identifies general environments in which the enforceable principle may or may not be violated.²⁶

²⁶Notably, my results hold both in the case that an external collector of fines may be appointed, and in the case that only budget-balanced contracts are allowed: This distinction is immaterial for my analysis.

As earlier promised, before I turn to this, I explore assumptions on the form taken by operators F representing verifiability constraints in absence of legal constraints, for t , s and $s' \in P(s)$ such that $t_{ij}(s) \neq t_{ij}(s')$, that are alternative to simply setting $F[t]_{ij}(s) = 0$. Of course maintaining that $F[t]_{ij}(s) = t_{ij}(s)$ when $t_{ij}(s) = t_{ij}(s')$ for all $s' \in P(s)$, there are several possibility with respect to the completion of the operator F . These different possibilities can be motivated by different rules on how Courts would ‘complete’ contracts when they cannot establish which transfers should be enforced, on the basis of the stipulated contracts.

One possibility, for example, is a ‘conservative completion rule’, by which $F[t]_{ij}(s) = \min_{s' \in P(s)} t_{ij}(s')$. In words, the Court is unsure about the precise commitment $t_{ij}(s)$ of i vis-a-vis j , but she is sure that i vis-a-vis j has committed to transfer at least $\min_{s' \in P(s)} t_{ij}(s')$ and hence she enforces that transfer. Another possibility would be that the Court does not take in consideration individual commitments t_{ij} separately from net transfers $n_{ij} = t_{ij} - t_{ji}$. The conservative rule can then also be defined for net transfers, setting $n_{ij}(s) | F[t] = \min_{s' \in P(s)} n_{ij}(s') | t$ when $n_{ij}(s') | t > 0$ for all $s' \in P(s)$, $n_{ij}(s) | F[t] = \max_{s' \in P(s)} n_{ij}(s') | t$ when $n_{ij}(s') | t < 0$ for all $s' \in P(s)$ and $n_{ij}(s) | F[t] = 0$ otherwise. The specific form of $F \circ t$ does not matter, here, as long as $F[t]_{ij}(s)$ is constant on $P(s)$.

A final possibility that I will consider in this paper is a ‘seniority rule’, by which the Court presumes that one player, i , is disadvantaged in case of illegal or unverifiable net transfers with respect to j . For example, if i is a principal and j is an agent, it is reasonable to presume that, in doubt, the Court will enforce the net transfer most favorable to j , so that I can set $n_{ij}(s) | F[t] = \max_{s' \in P(s)} n_{ij}(s') | t$. And, again the specific form of $F \circ t$ does not matter, here, as long as $F[t]_{ij}(s)$ is set constant on $P(s)$.

As pointed out earlier, the added value of my formalization goes beyond representing the constraint that courts do not enforce transfers that they cannot verify. Its main added value lies in the representation of legal rules that constrain what transfers can be legally enforced. We have already informally introduced two legal rules in section (2): individual liability and the damage compensation rule. In the next section, I show how to represent such legal rules within my formalization. Further, I use my formalization also to represent

the constraint of bounded, or limited liability, mentioned at the beginning of this section.

4 Analysis

4.1 Fundamental Results

This section provides general results on the mathematical structure of the operator F that relate to the concept of enforceable contracts, and to the enforceability principle. Then I show that this principle holds when all contracts are legal, if the Court's information structure which underlies the verifiability constraints is partitional, as it is usually assumed in contract theory. In other terms, under 'standard' assumptions, the enforceability principle holds, and there is no loss in restricting attention to enforceable contracts.

A first observation that would lead much of the ensuing analysis is that, in any game G , the concept of enforceable contract is intimately related with simple mathematical properties of the operator F . In fact, a contract t is enforceable whenever all its stipulated transfers t_{ij} coincide with the enforced transfers $F[t]_{ij}$, or $F \circ t = t$, i.e. whenever t is a *fixed point* of F . Likewise, for any strategy profile s , a contract t is enforceable at s whenever $F \circ t(s) = t(s)$.

On the basis of this observation, it is possible to derive mathematically workable necessary and sufficient conditions for the enforceability principle to hold in different legal environments. First, for any transformation operator F , I define the inverse operator F^{-1} as follows: For every contract t , the set $F^{-1}(t) = \{\hat{t} : F(\hat{t}) = t\}$ contains all stipulated contracts \hat{t} that will be enforced as t by Courts. Evidently, again t is enforceable if and only if t is a fixed point of the correspondence F^{-1} ; whereas for any outcome s , whenever $F^{-1} \circ t$ contains at least one contract \hat{t} such that $t(s) = \hat{t}(s)$, the contract \hat{t} is enforceable at s . Then, given a game G , for any outcome s , I define as $T(s)$ the set of *enforced contracts* t that achieves s , i.e., $T(s) = \{t : u_i(s) + n_i(s) \geq u_i(s'_i, s_{-i}) + n_i(s'_i, s_{-i}) \mid t, \text{ for all } i, \text{ and all } s'_i \in S_i\}$. Evidently, a strategy profile s can be achieved in game G under a legal rule \mathcal{L} that induces an operator F if and only if there exists a contract $t \in T(s)$ such that $F^{-1}(s)$

is non-empty.

Thus, we immediately obtain the two following general necessary and sufficient conditions for the enforceability principle. In these conditions, I make the distinction on whether an outcome s can be achieved with a contract enforceable at s , or with a contract enforceable at all strategy profiles $s' \in S$. The idea is that, when delivering a positive result, one would like to identify conditions so that the external modeller can restrict attention without loss to contract enforceable at all strategy profiles $s' \in S$; whereas, when finding a negative result, it is more important to identify an instance where a desirable strategy s cannot be achieved with any contract enforceable on path, when s is played, as in Examples 1 and 2.²⁷

Theorem 1 *For any game G and legal environment \mathcal{L} inducing the operator F ,*

1. *the enforceability principle holds if and only if, for any desirable strategy profile $s \in S$, it is the case that $F^{-1}(t)$ is empty for any $t \in T(s)$, or there exists $t \in T(s)$ such that $t \in F^{-1}(t)$;*
2. *for any desirable strategy profile $s \in S$, the enforceability principle holds at s if and only if it is the case that $F^{-1}(t)$ is empty for any $t \in T(s)$, or there exists $t \in T(s)$ such that $F^{-1} \circ t$ contains at least one contract \hat{t} such that $t(s) = \hat{t}(s)$.*

The mathematical workability of these results lies in translating the validity of the enforceability principle at any strategy profile s into the existence of fixed points of the inverse correspondence F^{-1} on the restricted domain $T(s)$. Standard mathematical results such as the Kakutani's fixed point theorem can then be used to establish the validity of the enforceability principle.

Further the above result the following simple (and slack) sufficient condition for the enforceability principle.

²⁷In the statement, I only consider contracts aimed at achieving outcomes s which I consider 'desirable', i.e., outcomes that are not Nash equilibria of the game, and yet are valuable to the players as, for example, they Pareto dominate all Nash equilibria.

Now, I use the above results to derive the very lax sufficient condition that if $F \circ F \circ t = F \circ t$ for all t , then all achievable outcomes can be achieved with a (globally) enforceable contract.

I note that this is equivalent to assume that F is idempotent, and that any \hat{t} such that $F \circ \hat{t} = \hat{t}$ is a fixed point of F .

Theorem 2 *Consider a game G and legal rule \mathcal{L} . If the induced operator F is idempotent, i.e. $F \circ F = F$, then any achievable outcome s can be achieved with an enforceable contract, so that the enforceability principle holds.*

Proof of Theorem 2. Suppose that F is idempotent, so that $F \circ F \circ t = F \circ t$ for all contracts t . Then, the contract $F \circ t$ is enforceable, by definition. Further any strategy profile s that can be achieved by a contract t can also be achieved with $F \circ t$, because $u + n|F \circ t = u + n|F \circ F \circ t$. Hence the enforceability principle holds for all $s \in S$. ■

The property of idempotence is known in several branches of mathematics, such as linear algebra, or group theory. Here, idempotence of F can be interpreted as follows. For the clarity of the argument, suppose that for any i, j , and s , the enforced transfer $F[t]_{ij}(s)$ can take only two values: $t(a)$ or zero; i.e., that the legal rule is such that Courts either enforce the stipulated transfers $t_{ij}(s)$ or they void them, setting $F[t]_{ij}(s) = 0$. Then, the operator F can be conceptualized as a ‘filter’: it enforces some transfers $t_{ij}(s)$ and voids others. Requiring that F is idempotent implies that, after court purges from the contract t all transfers $t_{ij}(s)$ that should be voided, a contract $F \circ t$ is obtained such that no further transfers should be voided.

This interpretation is especially stark in the case in which all outcomes are verifiable, i.e., $P = \{\{s\} : s \in S\}$. There, all that F represents is whether transfers $t_{ij}(s)$ are legal or not, within the context of contract t . Idempotence of F , thus, requires the following. After purging the contract t of all illegal transfers $t_{ij}(s)$, a contract $F \circ t$ is obtained such that no further transfers $F[t]_{ij}(s)$ should be purged. So, for example, idempotence of F is satisfied in all cases in which the enforced transfers $F[t]_{ij}(s)$ are independent of the form taken by

the stipulated transfers of $t(s')$ on any profile $s' \neq s$, i.e., whether a transfer $t_{ij}(s)$ should be enforced or not depends only on the “local” properties of the contract t at s , and not on the global characteristics of the contract t .

Reintroducing the possibility that some outcomes are not verifiable, consider the ‘standard’ case in which P is a (truthful) partition of S ; i.e., $s \in P(s)$ for all s , and for all $s' \in P(s)$, $P(s') = P(s)$. Most of contract theory implicitly assumes that all contracts are legal, or introduces bounded liability constraints motivated by the principle that individuals cannot be expropriated of fundamental inalienable rights. We now show that Theorem 2 implies that the enforceability principle holds generally when the only limitations to the enforcement of contracts are verifiability constraints, so that all contracts are legal, as well as when transfers are not enforced if they are excessively large.

Recall that, when all contracts are legal, the operator F is such that $F[t]_{ij}(s) = t_{ij}(s)$ if only if $t_{ij}(s') = t_{ij}(s)$ for all $s' \in P(s)$, and is otherwise determined by one of the completion rules described in the previous section. The additional constraint of bounded liability implies that $F[t](s) = t(s)$ if only if, for all i , $\min_{s' \in P(s)} u_i(s') + n_i(s) |t \geq \bar{u}_i$, for some given bounds \bar{u}_i with the property that $u_i(s) \geq \bar{u}_i$ for all s . In other terms, the Court must verify that no player i ’s utility falls below the level \bar{u}_i , which represents the ‘value’ of i ’s inalienable rights. When this is not the case, the court must enforce transfers $F[t](s)$ such that for all i , $\min_{s' \in P(s)} u_i(s') + n_i(s) |F \circ t \geq \bar{u}_i$. Again, one can introduce different rules completing F , as in section 3.²⁸ Remarkably, the following result holds for all these completion rules.

Proposition 1 *For any game G , and partitional verifiability structure P , absent legal constraints, or when the only constraint is bounded liability, any achievable outcome s can be*

²⁸For example, the avoidance rule, here, is such that $F[t](s) = 0$ unless for all $s' \in P(s)$, $t(s') = t(s)$ and for all i , $\min_{s' \in P(s)} u_i(s') + n_i(s) |t \geq \bar{u}_i$, the conservative rule is such that

$$F[t](s) = \arg \min_{\hat{t} \in \mathbb{R}^I \times \mathbb{R}^J, \min_{s' \in P(s)} u_i(s') + n_i(s) |\hat{t} \geq \bar{u}_i} \|\bar{t} - \hat{t}(s)\|, \quad (1)$$

where $\hat{t}_{ij}(s) = \min_{s' \in P(s)} t_{ij}(s')$ for all i, j ; whereas the seniority rule is such that that condition (1) holds for the transfers $\hat{t}(s)$ such that $n_{ij}(s) |\hat{t} = \max_{s' \in P(s)} n_{ij}(s') |t$, for all j more senior than i , and $\hat{t}_{ij}(s) = 0$ for any i, j such that $n_{ij}(s) |\hat{t} < 0$.

achieved with an enforceable contract, so that the enforceability principle holds.

The above result is very intuitive. When all transfers are legal, the only obstacle to enforcement is verifiability. The court will not enforce transfers $t(s)$ whenever t is not constant on $P(s)$. For clarity suppose that, in this case, the court voids the transfers and sets $F[t](s) = 0$. Evidently, then $F \circ t$ is constant on $P(s)$ either because $F[t](s)$ equals $t(s)$ which is constant on $P(s)$, or because $F[t](s') = 0$ for all $s' \in P(s)$. Thus, $F \circ F \circ t = F \circ t$, and F is idempotent. In other terms, when all transfers are legal and P is partitional, any change made by the operator F to transfers t make $F \circ t$ is constant on $P(s)$, and are independent of the form taken by t outside of $P(s)$. Thus, when F is applied to $F \circ t$, it does not induce further changes. In fact, the changes made by F to t outside $P(s)$ are irrelevant for how F should change $F \circ t$, and F has already turned t into a contract $F \circ t$ that is constant on $P(s)$, and hence conforms with the verifiability constraint.

The same logic holds when the only legal constraint is bounded liability. In this case, if F changes the transfers $t(s)$ associated to any profile s , it produces transfers $F[t](s)$ that are constant on $P(s)$ and such that $\min_{s' \in P(s)} u_i(s') + n_i(s) | F \circ t \geq \bar{u}_i$, so that $F \circ t$ automatically satisfies the bounded liability constraint, and F is idempotent.

4.2 The Role of Transitivity

This section shows how the positive result for the case in which all contracts are legal, presented in Proposition 1, can be generalized to court's information structures P that need not be partitional, identifies which properties of information are needed, and defines a form of transitivity which is key for the enforceability principle.

While partitional verifiability structures are often considered standard, it is worth investigating non-partitional structures, here. It is known that if a 'standard' player in a game knows her own information correspondence P , then P cannot be a partition. Suppose, for example, that $P(s) = \{s, s'\}$, whereas $P(s') = \{s'\}$. When s realizes, the player cannot exclude the possibility that s has taken place, whereas she knows that, had s' realized, she would have concluded that s did not take place. Hence, the argument goes, the player

concludes that s' did not take place, and can refine her information correspondence to $P(s) = \{s\}$ and $P(s') = \{s'\}$. In fact, Geanakoplos (1989) proved that truthful information correspondences P are partitional if and only if, whenever any outcome s realizes, the player knows what she would know and would not know had any other outcomes s' realized. As Shin (1993) pointed out, the requirement that an entity knows all that she does not know may be too stringent when knowledge of an event equates with the capability of proving that the event has occurred. Indeed, for a researcher is usually impossible to prove that she cannot prove a result. To the extent that a court is bound by rules of evidence similar to the ones adopted in scientific research, then, assuming that P is partitional may be too restrictive.

While previous work has focused on this ‘know-that-you-don’t-know’ axiom, this paper will uncover a crucial role for the requirement complementary that the court knows all that knows. This requirement is formally stated as *transitivity* of P : for any $a, b, c \in S$, $a \in P(b)$ and $b \in P(c)$ implies $a \in P(c)$. Our main positive result, Proposition 2, shows that if all contractual transfers are legal, and the court’s information correspondence P is transitive, then the enforceability principle holds, for all the contract completion rules presented in section ??

Proposition 2 *For any game G , and transitive verifiability structure P , absent legal constraints, the enforceability principle holds.*

This result is proved in the Appendix by invoking Theorem 2 and by showing that if all contractual transfers are legal, and the court’s information correspondence P is transitive, then the resulting operator F is idempotent. Here, we present a heuristic argument that highlights the role played by transitivity of P in ensuring that the enforceability principle holds. Proceeding by contradiction, we suppose that the enforceability principle fails. Specifically, we suppose that there is achievable outcome c , that cannot be achieved with any contract t by which the parties stipulate to play c , i.e. with any contract t such that $u_i(c) + n_i(c) |t \geq u_i(d_i, c_{-i}) + n_i(d_i, c_{-i}) |t$ for all player i and strategy d_i . So, there must be a player i who has a profitable deviation d_i such that player i cannot be deterred from

playing d_i when the opponents play c_{-i} and the players contractually committed to c . For this to happen, it must be the case that player i 's deviation d_i cannot be verified by the Court when the opponents play c_{-i} , i.e., it must be the case that $c \in P(d_i, c_{-i})$. Further, as the outcome c is achievable, there must exist a contract t with contractual transfers $n|t$ supposed to deter c (i.e., a contract t with which the parties “stipulate” to play a profile b different from c), but such that c is nevertheless a Nash Equilibrium when t is signed. So, it must be that the strategy c is not verified as a breach of contract when the players stipulated to play b , i.e., that $b \in P(c)$, and that player i is deterred from playing d_i when the opponents play c_{-i} and the players contractually committed to b , i.e., that $b \notin P(d_i, c_{-i})$. Hence, the verifiability structure P is shown not to be transitive, as $b \in P(c)$ and $c \in P(d_i, c_{-i})$, but $b \notin P(d_i, c_{-i})$.

We now further explore the role played by the transitivity condition of the verifiability structure P with respect to the enforceability principle. It is easy to formulate a simple example in which the enforceability principle does not hold, because the verifiability structure is intransitive. As anticipated earlier, the lack of transitivity of P is due to the fact that verifiability is costly. In fact, because verifying events is a costly activity, here, the cost of verifying that an event is verified may be socially excessive, so that the court's information correspondence fails to satisfy the ‘know-that-you-know’ axiom. Conversely, one can deem reasonable the ‘know-that-you-know’ axiom when court's verification is costless.

Example 3 (Verifiability with Costly Enforcement). Consider a simple multi-player continuous-strategy prisoner dilemma, in which each player i chooses $s_i \in [0, s_i^*]$, each player prefers to play a lower s_i , regardless of the opponents' choices s_{-i} , and the first-best outcome is s^* . Suppose that outcome verification and contract enforcement are costly. When player i 's plays s_i , a court may only legally rule that i 's choice belongs to the interval $[s_i - M, s_i + M]$, but is unable to determine precisely i 's choice in the set. The function can be interpreted $M > 0$ is a legal margin of error that could be made equal to zero only with a socially excessive expenditure in verification and enforcement by the legal system. The induced verifiability correspondence is not transitive. Consider three actions by player i , $c_i < b_i < a_i$, such that $a_i - b_i < M$ and $b_i - c_i < M$, but $c_i - a_i > M$. Then, evidently,

$a_i \in P(b_i, x_{-i})$ and $b_i \in P(c_i, s_{-i})$, but $a_i \notin P(c_i, s_{-i})$, regardless of the opponents' choices s_{-i} .

So, if player i commits to play the socially optimal choice s_i^* (i.e., to paying a sufficiently large penalty, to an external enforcer if found playing $s_i < s_i^*$), any deviation $s_i \in (s_i^* - M, s_i^*)$ cannot be deterred: The Court cannot establish that i did not play s_i^* , because $s_i^* \in [s_i - M, s_i + M]$. Due to the lack of transitivity in the court's information structure, the only contract that achieves the first-best s^* is the one where each player i commits to play $s_i^* + M$, and then violates the contract and plays s_i^* without being punished. The first-best outcome is achieved only with contracts that explicitly prohibits the first-best. \diamond

Example 3 formalizes the intuition that when enforceability limitations or verifiability costs make it impossible to deter only small deviations from the precise terms of a contract, it may be optimal to agree to an overdemanding contract, and achieve the first-best by allowing small breach of contracts. For instance, casual observation suggests that in some areas of the US it is considered unlikely to be sanctioned when violating the speed limit by less than 10 M.p.h. on a highway. In fact, in the faster lanes the traffic often flows at a speed that is roughly 10 M.p.h. above the limit. Furthermore, many drivers believe that this is justifiable because speed limits are too restrictive. Now, suppose that the socially optimal top speed on a highway is x M.p.h., but it is not practically feasible to sanction speed violations unless they are at least 10 M.p.h. above the speed limit. For example, drivers could challenge small speed limit violations by claiming that they could not precisely ascertain their speed, or by challenging the Police reports, and so on and so forth. The administrative costs sustained by the Courts to settle all these debatements would be socially excessive. It would be more cost effective that enforcement officers exert leeway for speed limit violations of less than 10 M.p.h., and that the social contract crafted by the legislator incorporates this enforcement limitation, by setting the speed limit at $x - 10$ M.p.h. As a result of this simple and cost effective policy, the faster traffic would flow at x M.p.h, the optimal speed. Most importantly for this paper, the use of this policy constitutes a violation of the enforceability principle.

Both Example 3 and the heuristic argument presented after the statement of Proposition 2 described the role played by the intransitivity of the Court’s information structures P to generate failure of the enforceability principle, when all contractual transfers are legal. In this case, the only enforcement constraint is that stipulated transfers be verifiable. When P is intransitive, it may be that the breach (d_i, c_{-i}) of a contract stipulating the first-best outcome c is not verifiable, but that parties can achieve first best with a contract that ‘stipulates’ a different outcome b , because the breach (d_i, c_{-i}) of that second contract is verifiable, whereas the breach c is not.

When it is not the case that all transfers are legal, the enforceability principle may fail also when the Court’s information structures P is transitive, or even partitional, as shown by Examples 1 and 2. Nevertheless, it is still possible to identify a role for transitivity of a suitably defined order in generating instances in which the enforceability principle holds. In heuristic terms, failure of the enforceability principle is generated by a failure of “transitivity of incentives” which can be described informally as follows: “Contractually stipulating b deters the play (d_i, c) but not c , whereas contractually stipulating c does not deter the play (d_i, c) .”

Formally, given the game $G = (I, S, u)$, and the operator F , I define the following “legal incentive order” as follows: $(c_i, b_i) \in L_i[a_{-i}, u, F]$, or $c_i L_i[a_{-i}, u, F] b_i$, whenever there exist contract t such that

$$n_i(b_i, a_{-i})|F[t] \leq n_i(c_i, a_{-i})|t + u_i(c_i, a_{-i}) - u_i(b_i, a_{-i}). \quad (2)$$

In English, the notation $(c_i, b_i) \in L_i[a_{-i}, u, F]$ means that ‘given that the other players play a_{-i} in game G , it is legally feasible for player i to not deviate to b_i when contractually committing to play c_i ’. The importance of this “legal incentive order” is that the failure of its negative transitivity generates a failure of the enforceability principle at some outcome a , for the cases of interest in which a is achievable in the first place, as we now state precisely.

Proposition 3 *For any $G = (I, S, u)$ and F , if there exists a player i and a profile a_{-i} such that the legal incentive order $L_i[a_{-i}, F, u]$ fails to be negative-transitive, then, either a*

is not achievable, or the enforceability principle fails on path at a .²⁹

This result shows how the role played by intransitivity in generating failures of the enforceability principle, identified as intransitivity of the verifiability structure P in absence of legal constraints to contract enforcement, can be generalized to legal environments where both legal and verifiable constraints may limit contractual enforcement. In this general environment, the role of intransitivity is identified by failure of negative transitivity of the ‘legal incentive order’ $L_i[a_{-i}, u, F]$. Interestingly, because $L_i[a_{-i}, u, F]$ is a strict and complete preference order, for any a_{-i}, u and F , a failure of negative transitivity of $L_i[a_{-i}, u, F]$ is a manifestation of failure of failure of “rationality” for that order (in the sense of Savage).

Having completed the derivation of our results characterizing general conditions for the enforceability principle to hold or fail, the next section is devoted to revisit our motivating economic examples 1 and 2 in light of the general results derived here.

5 Revisitation of the Economic Examples

The previous section identified necessary and sufficient conditions for the enforceability principle to hold. Importantly, I established that this is the case when verifiability structure P is partitional, as long as all contracts are legal (or, at least, as long as the only legal constrain is bounded liability). In our motivating Examples 1 and 2, the enforceability principle is shown to fail, despite the fact that the verifiability structure is partitional, because of two legal constraints, individual liability and the damage compensation rule, respectively, that limit the enforcement of contractual transfers. We now return to revisit examples 1 and 2 to explain in depth the source of the failure of the enforceability principle, in relation with the formalization introduced in section ?? and the results of section 4.

We begin by reconsidering example 1, where the failure of enforceability principle is due to the fact that agents are only individually liable, and yet the production of a good

²⁹And it is also the case that the enforceability principle always fails for some u' and F' , possibly different from u and F , that conform with u and F at a , in the sense that there exists a contract t that achieves a for $G = (I, S, u')$ and F' and such that $u'(a) + n|F'[t](a) = u(a) + n|F[t](a)$.

quality good requires that both exert effort.

Individual Liability Rule and Example 1 Revisited. When players are only individually liable, the court will sanction player i only if it identifies her as a contract violator. Suppose, for simplicity, that the Court adopts the ‘voidance contract completion rule’ by which, for any i, j and s , it is the case that the implemented transfer $F[t]_{ij}(s)$ either equals the stipulated transfer $t_{ij}(s)$ or zero.³⁰ First, the court voids any contractual transfer scheme t where the committed transfer by a player i may depend on the opponents’ actions: In any such a contract t , the individual commitment of player i cannot be logically identified. Formally $F[t](s)$ takes the value of zero for all strategy profiles, if there is a pair of players i, j such that $t_{ij}(s)$ is not constant in s_{-i} for some s_i . Hence, the only contracts t that may have some strategic effect are such that the transfers $t_{ij}(s_i, s_{-i})$ are constant on s_{-i} .

Second, the court will void any committed transfer $t_{ij}(s)$ by a player i to any player j , whenever she cannot verify the individual contractual commitment of player i . Formally, for any set of outcomes $S' \subseteq S$, let the set $\pi_i(S') = \{s_i \in S_i \mid (s_i, s_{-i}) \in S' \text{ for some } s_{-i} \in S_{-i}\}$ denote the *projection* of S' onto S_i . We then stipulate that for any outcome s , the enforced transfer by player i to player j is voided, i.e., $F[t]_{ij}(s) = 0$, whenever there exists a player i ’s strategy $s'_i \in \pi_i(P(s))$ such that $t_{ij}(s_i) \neq t_{ij}(s'_i)$, where we drop the argument s_{-i} from t_i , for brevity. In words, the Court enforces the transfer $t_{ij}(s)$ only if it verify the individual commitment of player i , i.e., only if she can conclude that player i played a strategy for which she committed to transfer $t_{ij}(s_i)$. If the projection $\pi_i(P(s))$ of what she can verify, $P(s)$, onto player i ’s choice set S_i includes also actions s'_i for which i committed to a different transfer, then the Court does not enforce the transfer $t_{ij}(s_i)$.

Now, reconsider Example 1. Suppose that each one of the two agents commits to pay the principal a penalty unless she works hard, without committing to monitor the other agent. The first best cannot be achieved in equilibrium. Even if agent 2 works hard without monitoring, when agent 1 shirks, her penalty cannot be enforced by the court under

³⁰It can be verified that all our results, here, hold also if the Court adopts a ‘conservative completion rules’ constructed in the same way as the analogous rule described in section 3.

individual liability. In fact, the realized outcome is (LO, HO) , hence the court's information is the set $P(LO, HO) = \{(LM, HO), (LO, HO), (LO, LO), (HO, LO), (HO, LM)\}$, and the projection $\pi_1(P(LO, HO)) = \{HO, LO, LM\}$ onto player 1's action space includes the action HO .

Suppose now instead that each one of the two agents commits to play HM , i.e. work hard and monitor the other agent. Now, the first best can be achieved in equilibrium. In fact, if agent 2 works hard without monitoring agent 1, and agent 1 shirks, the projection $\pi_1(P(LO, HO)) = \{HO, LO, LM\}$ onto player 1's action space does not include the action HM . Hence the penalty that agent 1 contracted to pay will be enforced by the court.

While the verifiability structure P is partitional, and hence transitive, the failure of the enforceability principle displayed in Example 1 can be understood in relationship with Proposition 3. With the operator F defined here, in fact, the legal incentive order $L_i[a_{-i}, u, F]$ takes the simple form:

$$(c_i, b_i) \in L_i[a_{-i}, u, F] \text{ if and only if } c_i \notin \pi_i(P(b_i, a_{-i}));$$

in words, given that the opponents play a_{-i} , the court can conclude that player i did not play c_i , if i played b_i . Consider Example 1: because $HO \in \pi_i(P(HM, HO))$ and $LO \in \pi_i(P(HO, HO))$ but $LO \notin \pi_i(P(HM, HO))$, for both $i = 1, 2$, it follows that $(HO, HM) \notin L_i[HO]$, $(LO, HO) \notin L_i[HO]$, but $(LO, HM) \in L_i[HO]$, so that the order $L_i[HO]$ fails to be negative transitive, and Proposition 3 applies. In other terms, each one of the agents can be deterred for playing LO when contractually committing to play HM , as long as the other one plays HO , whereas she cannot be deterred from playing LO when committing to play HO , nor for playing HO when committing to play HM . \diamond

Now, we turn to reconsider example 2, where the failure of the enforceability principle is due to the fact that punitive transfers are not enforced, and that the principal cannot verify that she suffered damages due to the agent's breach of contractual commitment, unless the parties agree to overspecify the contract.

Damage Compensations and Example 2 Revisited. Suppose that punitive transfers

are not enforced, so that transfers are limited to the compensation for the verified damages incurred because of a breach of contract. Also, because here player 2 is the principal of player 1, it is reasonable to suppose that contracts are completed according to the seniority rule favorable to player 1.

To define this seniority rule, I proceed as in section 3. Simplifying notation slightly, let $n_2(b)|t$ be the net transfer from the principal, player 2, to the agent, player 1, induced by a contract t , i.e., the payment for the good agreed under t for the contingency b . Under the damage compensation rule, the transfer $n_2(b)|t$ to the agent is implemented at b if and only if it is the case that for any outcome c , and any outcome $d \in P(b)$, the difference in payment $[n_2(c)|t - n_2(b)|t]$ is weakly smaller than the principal's utility loss $\max\{0, u_2(c) - u_2(d)\}$. In fact, if the realized contingency is b , the court only knows that an outcome in $P(b)$ but cannot distinguish which one occurred. Supposing that the contract has been written with the agreement that c should be played, and the principal should pay the agent $n_2(c)|t$, the Court cannot allow that the principal reduces the payment to the amount $n_2(b)|t$, if the difference in payment $[n_2(c)|t - n_2(b)|t]$ is larger than the principal's utility loss verified by the Court. The actual principal's utility loss is $\max\{0, u_2(c) - u_2(b)\}$, but because the Court does not know which outcome $d \in P(b)$ precisely occurred, she cannot allow the payment $n_2(b)|t$, whenever there exist a profile $d \in P(b)$ such that $n_2(c)|t - n_2(b)|t > \max\{0, u_P(c) - u_P(d)\}$. Finally, for the construction to be internally consistent, this constraint needs to hold for all possible outcomes c .

Hence, the implemented net transfers $n_2(b)|F[t]$ must follow the rule $n_2(c)|t - n_2(b)|F[t] \leq \max\{0, u_2(c) - u_2(d)\}$ for all $d \in P(b)$; or $n_2(c)|t - n_2(b)|F[t] \leq \min_{d \in P(b)} \max\{0, u_2(c) - u_2(d)\}$ for all c . Or, $n_2(b)|F[t] \geq n_2(c)|t - \min_{d \in P(b)} \max\{0, u_2(c) - u_2(d)\}$ for all c , or $n_2(b)|F[t] \geq \max_{c \in S} \{n_2(c)|t - \min_{d \in P(b)} \max\{0, u_2(c) - u_2(d)\}\}$. Hence, completing the operator F with the seniority rule, we let

$$n_2(b)|F[t] = \max \left\{ n_2(b)|t, \max_{c \in S} \left\{ n_2(c)|t - \min_{d \in P(b)} \max\{0, u_2(c) - u_2(d)\} \right\} \right\}. \quad (3)$$

We now return to example 2, and we simplify the exposition, without loss of generality, by focusing on contracts t such that the principal's net transfer to the agent $n_2|t$ can be

decomposed as follows. For any effort choice \mathbf{e} by the agent, $n_2(\mathbf{e})|t = t_2 - t_1(\mathbf{e})$, where $t_2 \geq 0$ denotes a fixed payment from the principal to the agent, whereas $t_1(\mathbf{e}) \geq 0$ is a penalty paid by the agent to principal in relation to possible breach of contracts. Hence, we reformulate our definition (3) as follows:

$$F[t]_1(\mathbf{e}) = \min \left\{ t_1(\mathbf{e}), \min_{\mathbf{e}' \in \{0,1\}^2} \left\{ t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\} \right\} \right\}. \quad (4)$$

In other terms, the Court enforces the stipulated penalty $t_1(\mathbf{e})$ only when it is not larger than $\min_{\mathbf{e}' \in \{0,1\}^2} \{t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\}\}$, i.e., when $t_1(\mathbf{e})$ is not larger than $t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\}$ for any possible agent's choice $\mathbf{e}' \in \{0,1\}^2$. If, in fact, it is the case that $t_1(\mathbf{e}) - t_1(\mathbf{e}')$ is larger than $\min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\}$, then the 'excess penalty' $t_1(\mathbf{e}) - t_1(\mathbf{e}')$ paid by the agent when playing \mathbf{e}' instead of \mathbf{e} is not justified by the verified utility loss $\min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\}$ suffered by the principal because the agent played \mathbf{e}' instead of \mathbf{e} .

Before entering starting the formal analysis of example 2, let me note that, for any agent's choice $\mathbf{e} \in \{0,1\}^2$, the maximal principal profit according to the Court's information $\max_{\mathbf{e}'' \in P(\mathbf{e})} u_2(\mathbf{e}'') = v(\mathbf{e}_A + 1)$, because the task B is not verifiable.

Now, consider the contract t is such that $t_1(\mathbf{e}) = 0$ if $e_A = 1$ and $e_B = 1$, $t_1(\mathbf{e}) = v(2) - v(1)$ if $e_A + e_B = 1$, and $t_1(\mathbf{e}) = v(2) - v(1)$ if $e_A + e_B = 0$. We show that $F[t]_1(\mathbf{e}) = t_1(\mathbf{e}) = v(2) - v(1)$ when $e_A = 0$ and $e_B = 0$; and that $F[t]_1(\mathbf{e}^*) = 0$ for $e_A^* = 1$ and $e_B^* = 0$; so that player 1 optimally chooses \mathbf{e}^* , for which $0 = F[t]_1(\mathbf{e}^*) \neq t_1(\mathbf{e}^*) = v(2) - v(1)$. Hence, contract t achieves the outcome \mathbf{e}^* but fails to be enforceable.

First, consider \mathbf{e} such that $e_A = 0$ and $e_B = 0$, so that $t_1(\mathbf{e}) = v(2) - v(1)$ and note that this is smaller than $t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\}$ both when $e'_A = 1$ and $e'_B = 1$ and when $e'_A + e'_B = 1$. In the first case, in fact, $t_1(\mathbf{e}') = 0$, $u_2(\mathbf{e}') = v(2)$, and $\max_{\mathbf{e}'' \in P(\mathbf{e})} u_2(\mathbf{e}'') = v(1)$, so that $t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\} = v(2) - v(1)$, using the equality $\min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\} = \max\{0, u_2(\mathbf{e}') - \max_{\mathbf{e}'' \in P(\mathbf{e})} u_2(\mathbf{e}'')\}$. When $e'_A + e'_B = 1$, instead $t_1(\mathbf{e}') = v(2) - v(1)$ and $u_2(\mathbf{e}') = v(1)$, so that $t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\} = v(2) - v(1)$, again. Hence, $F[t]_1(\mathbf{e}) = t_1(\mathbf{e}) = v(2) - v(1)$ when $e_A = 0$ and $e_B = 0$.

Second, consider $\mathbf{e} = \mathbf{e}^*$ such that $e_A^* = 1$ and $e_B^* = 0$. Again, $t_1(\mathbf{e}) = v(2) - v(1)$, but here, $\min_{\mathbf{e}' \in \{0,1\}^2} \{t_1(\mathbf{e}') + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}') - u_2(\mathbf{e}'')\}\} = 0$, as it is shown by taking \mathbf{e}' such that $e'_A = 1$ and $e'_B = 1$, so that $t_1(\mathbf{e}') = 0$ and $u_2(\mathbf{e}')$, noting that $\max_{\mathbf{e}'' \in P(\mathbf{e})} u_2(\mathbf{e}'') = v(2)$, here. Hence $0 = F[t]_1(\mathbf{e}^*) \neq t_1(\mathbf{e}^*) = v(2) - v(1)$.

Having concluded that the above specified contract t achieves the optimal outcome \mathbf{e}^* such that $e_A^* = 1$ and $e_B^* = 0$, but fails to be enforceable, we now proceed to show that there is no contract t that achieves \mathbf{e}^* and that is enforceable on the equilibrium path. Suppose by contradiction, in fact, that t is enforceable at \mathbf{e}^* , i.e., $F[t]_1(\mathbf{e}^*) = t_1(\mathbf{e}^*)$, and consider \mathbf{e} such that $e_A = 0$ and $e_B = 0$. Noting that $\max_{\mathbf{e}'' \in P(\mathbf{e})} u_2(\mathbf{e}'') = v(1)$, here, we obtain that $t_1(\mathbf{e}^*) + \min_{\mathbf{e}'' \in P(\mathbf{e})} \max\{0, u_2(\mathbf{e}^*) - u_2(\mathbf{e}'')\} = t_1(\mathbf{e}^*) + v(1) - v(1)$, and hence that $F[t]_1(\mathbf{e}) \leq t_1(\mathbf{e}^*)$, using definition (4). As the enforced penalty for shirking and playing $e_A = 0$ and $e_B = 0$ cannot be larger than the enforced ‘penalty’ for playing the optimal effort choice $e_A^* = 1$ and $e_B^* = 0$, it follows that t cannot achieved \mathbf{e}^* .

Again, this example can be explained in terms of a failure of (negative) transitivity in line with Proposition 3. Intuitively, while the damage incurred by player 2 cannot be verified if player 1 committed to action \bar{e} such that $\bar{e}_A = 1$ and $\bar{e}_B = 1$ and played $e_A^* = 1$ and $e_B^* = 0$, and the damage incurred by player 2 cannot be verified if player 1 committed to action $e_A^* = 1$ and $e_B^* = 0$ and played $\underline{e}_A = 0$ and $\underline{e}_B = 0$, the damage is verified when player 1 committed to $e_A = 1$ and $e_B = 1$ $\bar{e}_A = 1$ and $\bar{e}_B = 1$ and played $\underline{e}_A = 0$ and $\underline{e}_B = 0$. Formally, the legal incentive order $L_1[s_2, u, F]$ can be constructed using definitions 2 and 4, here, to show that $(\bar{\mathbf{e}}, \mathbf{e}^*) \notin L_1[B]$ and $(\mathbf{e}^*, \underline{\mathbf{e}}) \notin L_1[B]$, but $(\bar{\mathbf{e}}, \underline{\mathbf{e}}) \in L_1[B]$. \diamond

6 Conclusion

Real-world transactions are often limited: Not all contracts are legal. As a result, I have shown economic examples where the only way to achieve first-best is as follows. First, the parties contractually commit to an inefficient different outcome. Then, they secretly violate the contract and coordinate play on first-best. If the parties were to contractually commit to first best, they would fail to achieve it. So, first best can only be achieved by signing a

contract that appears to be over-specified, as it contains unenforceable prescriptions.

Motivated by these economic examples, I have developed a novel and general theory of how contracts are shaped by legal constraints. Instead of assuming that parties only sign enforceable contracts, the model developed here specifies what would happen in court when the signed contract includes unenforceable contingent-transfers, possibly distinguishing across different contingencies. In this paper, I use this novel framework to ask: When is the assumption that parties sign only enforceable contracts without loss? I have shown that this is the case, if all contracts are legal, under standard assumptions about verifiability. Within this standard domain, usual contract-theoretical modelling is without loss. But my richer formalization identifies two legal principles because of which the enforceability principle may fail in economically relevant examples. The first one, ‘individual liability, is the principle that a single individual may not always be sanctioned for a collective breach of contract, unless it is verified that she is among those who violated her contractual commitment. The second principle, ‘damage compensation’, here, prescribes that, even if it is verified that one breached contract, she cannot be legally forced to pay her counterparts more than their verified damages.

As the value of this paper is largely provocative, there are many possibilities for future research. As it uncovers surprising implications of the simple observation that not all contingent transfers that can be possibly included in a contract are legal, this paper may provide new ideas for contract theoretical studies of the implications of legal limitations to enforceability. In particular, the legal principles that generate the economic examples presented here (individual liability and damage compensation) are somewhat underexplored, to my knowledge. Further, the general theoretical model presented here is sufficiently broad that it may be used to ask questions entirely different from the object of this analytical enquiry (i.e., whether the enforceability principle holds). Finally, already noted in footnote 25, it would be interesting to expand the theoretical model presented here to account for the possibility of stochastic signals verifiable in court, and to move beyond the simple environment of strategic form games studied here, to allow for incomplete information and explicit modelling of sequential moves in extensive form games.

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8 Appendix

Proof or Proposition ??. Suppose that all transfers are legal. For any contract t , the contract $F \circ t$ is such that $F[t](s) = t(s)$ whenever t is constant on $P(s)$, and $F[t](s) \neq t(s)$ otherwise. Consider any contract completion rule such that the value of $F[t](s)$, when different from $t(s)$, is constant on $P(s)$. Then, evidently, $F[t](s)$ is constant on $P(s)$ for all s , so that $F[F[t]](s) = F[t](s)$. Clearly, the voidance rule, as well as the other rules introduced in section 3 are such that $F[t](s)$ is constant on $P(s)$ even when different from $t(s)$.

Suppose that the only legal constraint is bounded liability, with bounds \bar{u}_i for all i . Here, $F[t](s) = t(s)$ if and only if $t(s)$ is constant on $P(s)$ and $\min_{s' \in P(s)} u_i(s') + n_i(s)|t \geq \bar{u}_i$. When this is not the case, again, $F[F[t]](s) = F[t](s)$ for any completion rule such that $F[t](s)$ is constant on $P(s)$ even when different from $t(s)$. And, again, this is the case for all the completion rules defined in footnote 28. ■

Proof of Proposition 2. I first show that if all contracts are legal, and P is transitive, then F is idempotent.

By contradiction, suppose that there is a contract t such that $F \circ F \circ t(a) \neq F \circ t(a)$. I.e., there is i, j such that $F[F[t]]_{ij}(a) \neq F[t]_{ij}(a)$ for some a . Under the avoidance rule, for this to be the case, it must be that $0 = F[F[t]]_{ij}(a) \neq F[t]_{ij}(a) = t_{ij}(a)$. The second equality implies that, for all $s \in P(a)$, it is the case that $t_{ij}(s) = t_{ij}(a)$. The inequality implies that there exists $b \in P(a)$ such that $F[t]_{ij}(b) \neq F[t]_{ij}(a)$. Because $F[t]_{ij}(a) = t_{ij}(a)$ and, as concluded above, $t_{ij}(b) = t_{ij}(a)$, this means that $F[t]_{ij}(b) \neq t_{ij}(b)$. Hence, there must be $c \in P(b)$ such that $t_{ij}(b) \neq t_{ij}(c)$. But we earlier found that $t_{ij}(s) = t_{ij}(a)$ for all $s \in P(a)$; hence it must be that $c \notin P(a)$, and we have derived a failure of transitivity.

Turning to the conservative rule, in which $F[t]_{ij}(a) = \min_{d \in P(a)} t_{ij}(d)$, of course, in the case that $F[t]_{ij}(a) = t_{ij}(a)$, the proof is done. So, continuing to show the contradiction, suppose that $F[t]_{ij}(a) \neq t_{ij}(a)$ so that $\min_{d \in P(a)} t_{ij}(d) \neq t_{ij}(a)$. Hence, there must be a $b \in P(a)$ such that $F[t]_{ij}(b) < F[t]_{ij}(a) = \min_{d \in P(a)} t_{ij}(d)$. Because $F[F[t]]_{ij}(a) \neq F[t]_{ij}(a)$, it also follows that $\min_{d \in P(a)} F[t]_{ij}(d) \neq F[t]_{ij}(a)$. So, there must be a $c \in P(b)$ such that $t_{ij}(c) < F[t]_{ij}(a) = \min_{d \in P(a)} t_{ij}(d)$ and that $t_{ij}(c) < t_{ij}(b)$. But if $t_{ij}(c) < F[t]_{ij}(a)$, then it must be the case that $c \notin P(a)$. So, we have a failure of transitivity, as $c \in P(b)$, $b \in P(a)$ and $c \notin P(a)$.

It is easy to see that the same logic establishes the claimed result also for the other rules defined in section 3. ■

Proof of Proposition 3. Simplifying notation, suppose that $(a_i, b_i) \notin L_i[a_{-i}]$, so that $n_i(b_i, a_{-i})|F[t] > n_i(a_i)|t + u_i(a_i) - u_i(b_i, a_{-i})$ for any contract t . Hence, for any contract t such

that $F[t]_i(a) = t_i(a)$, it is the case that $n_i(b_i, a_{-i})|F[t] + u_i(b_i, a_{-i}) > n_i(a)|F[t] + u_i(a)$. In other terms, a cannot be achieved with any contract enforceable at a . Suppose further that $(c_i, a_i) \notin L_i[a_{-i}]$ and that $(c_i, b_i) \in L_i[a_{-i}]$, and note that this violates negative transitivity for $L_i[a_{-i}]$. By definition, there exists a contract t such that $n_i(a)|F[t] > n_i(c_i, a_{-i})|t + u_i(c_i, a_{-i}) - u_i(a)$ and $n_i(b_i, a_{-i})|F[t] \leq n_i(c_i, a_{-i})|t + u_i(c_i, a_{-i}) - u_i(b_i, a_{-i})$. These inequalities imply that $n_i(b_i, a_{-i})|F[t] - F[t]_i(a) \leq n_i(c_i, a_{-i})|t + u_i(c_i, a_{-i}) - u_i(b_i, a_{-i}) - [n_i(c_i, a_{-i})|t + u_i(c_i, a_{-i}) - u_i(a)]$, or, $n_i(b_i, a_{-i})|F[t] - n_i(a)|F[t] \leq -u_i(b_i, a_{-i}) + u_i(a)$, or, $n_i(b_i, a_{-i})|F[t] + u_i(b_i, a_{-i}) \leq n_i(a)|F[t] + u_i(a)$. Thus, the contract a achieves the desirable outcome a , and the enforceability principle is violated. ■