

Location as an Asset*

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Abstract

The location of individuals determines their job opportunities, living amenities, and housing costs. We argue that it is useful to conceptualize the location choice of individuals as a decision to invest in a ‘location asset’. This asset has a cost equal to the location’s rent, and a payoff through better job opportunities and, potentially, more human capital for the individual and her children. As with any asset, savers in the location asset transfer resources into the future by going to expensive locations with good future opportunities. In contrast, borrowers transfer resources to the present by going to cheap locations that offer few other advantages. As in a standard portfolio problem, holdings of this asset depend on the comparison of its rate of return with that of other assets. Differently from other assets, the location asset is not subject to borrowing constraints, so it is used by individuals with little or no wealth that want to borrow. We provide an analytical model to make this idea precise and to derive a number of related implications, including an agent’s mobility choices after experiencing negative income shocks. The model can rationalize why low wealth individuals locate in low income regions with low opportunities even in the absence of mobility costs. We confirm the core predictions of our theory with French individual panel data from tax returns.

1 Introduction

Few decisions determine an individual’s life more than the location decision. It determines job opportunities, social interactions, schooling and entertainment options, as well as a number of other less central characteristics of someone’s life. The location decision is essential particularly because of the large heterogeneity in location characteristics, even within a country, a state, a region, or a city. Living in Soho in Manhattan is quite different than living in Queens, and a world apart from living in parts of Newark or Camden, New Jersey. These spatial differences are enormous. Life prospects for a kid growing in Palo Alto are staggeringly different than those for someone growing in central Detroit, even if they come from similar backgrounds and

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both go to local public schools. The obvious question that arises is then, why do people remain in some of these locations? Why do we fail to see people go to the locations that offer the best prospects for them and for their families?

Three main answers have been offered to these questions in the economics literature. The first one relies on the presence of large migration costs that make moving to better locations not worth the cost.¹ The second one argues that local living costs, as reflected in housing and other local prices, compensate for other local benefits over the residency period. The third one simply says that agents ‘cannot afford’ to live in some places perhaps due to indivisibilities in housing. The problem with the first explanation is that it is hard to imagine that moving costs are sufficient to bridge the gap between the best and worse neighborhoods in virtually all regions of the world. These largely unobserved costs seem to be just a stand-in for another mechanism. As for the other two explanations, although housing and other local costs can differ substantially across regions, adjusting the size of one’s apartment, commuting from cheaper locations, and buying in big-boxed stores and other national retailers are effective strategies to deal with local prices.² Something is missing from this basic notion of static spatial equilibrium where similar marginal movers equalize utility across locations adjusted for moving costs.

In this paper we propose a different way of conceptualizing the location decision of agents. We argue that the location decision can be understood as an asset investment decision. Buying more of the asset involves moving to better locations that cost more but give better returns, while selling the asset implies moving to cheaper locations with little opportunities. The ‘Location as an Asset’ view can explain why agents prefer locations that seem undesirable from a static spatial equilibrium perspective even in the absence of moving costs. It can also explain why local living costs compensate the benefits from desirable locations for some agents but not for others, even in the absence of non-homotheticities or differences in preferences. The ‘location asset’ should not be confused with ‘an asset at a location’, like a house. The location asset is used by all agents, including renters and owners, when they make location choices.

The location asset has some specific features that make it different than other assets and determine their use. As any other asset, unconstrained agents own it only to the extent that the return from doing so dominates that of other assets, in particular risk free bonds. The key characteristic of the location asset is that it is not subject to borrowing constraints. Agents can always borrow, namely, transfer resources from the future to the present by going to cheaper location with worse opportunities. As long as they are not in the worst possible location already they can keep transferring resources from the future to the present, that is, sell the locations asset. The other key characteristic is that the amount of the asset that they can hold is limited by the housing needs, labor supply, fertility decisions and other choices that determine the current cost and the future benefits of living in a particular location. As such, the asset has heterogeneous

¹Kennan and Walker (2011) estimate that moving costs as large as \$380 thousand 2010 dollars (for young movers, 312 thousand for average ones) are needed to account for observed migration flows using a state-of-the-art model of location decisions. Diamond, McQuade, and Qian (2017) using a policy that implements rent-controls in the San Francisco area find a smaller but still large fixed cost of around \$40 thousand.

²Another potential reason for these location choices are non-homotheticities in preferences: the less wealthy simply like certain amenities better and the locations that have them are the ones with worse opportunities.

returns depending on the holder of the asset.

Conceptualizing location decisions as buying and selling a ‘location asset’ is useful to understand mobility decisions. Consider an agent with little or no wealth that receives a front-loaded income shock. For example, a blue-collar worker in the automobile industry in Detroit that gets fired. Where will she go? A good neighborhood with excellent schools for her children and plenty of job opportunities or a run-down neighborhood in Saint Louis? Think first about the consumption-savings decision of this agent. The front-loaded shock makes her want to transfer consumption from the present to the future. In the absence of accumulated wealth, smoothing consumption requires borrowing. The absence of collateral, however, implies that she will be constrained to borrow using standard financial assets. What is left is to borrow using the location asset and downgrade to a cheaper location with worse opportunities. Constrained agents that receive bad shocks will have a higher demand for locations that offer few opportunities at minimal cost. Similarly, front-loaded positive shocks will make constrained individuals upgrade location so as to save using the location asset.

Multiple implications follow. For example, changes in the rewards for particular occupations will result in front-loaded shocks for dynastic families, since heads-of-households have already invested in an occupation while their descendants have yet to choose. Hence, these changes in rewards will lead to spatial segregation as auto workers who borrow locate in Detroit and computer programmers and Yoga teachers who save locate in Palo Alto. It also implies that low rates of return in financial market (e.g. low interest rates) result in low rates of return of the ‘location asset’ and therefore larger price differentials across locations, reminiscent of the low interest rate period after the 2008 financial crisis where some of the differentials in house prices increased.

To make precise our conceptualization of the location decision as an asset that does not face borrowing constraints, we start by proposing a simple two period economy where agents have heterogenous assets, incomes, and levels of skills. Agents have access to a risk free bond but face ad-hoc borrowing constraints that prevent them from borrowing beyond an exogenous amount. Individuals choose a consumption profile and a location, which in turn determines their current rent and income next period as a function of their skill. There is a continuum of locations that differ in the marginal return of a unit of skill. In equilibrium wealthy agents locate in their ideal city conditional on their skill, while constrained individuals, either because they have low assets levels or back-loaded incomes, locate in cities that pay less but where rents are lower. Namely, they borrow using the location asset. Back and front-loaded shocks have the effects described above.

We then present a fully-fledged infinite horizon dynamic model with similar characteristics in order to generalize our findings and provide a framework that is potentially closer to quantitative analysis. Agents now face an idiosyncratic income process. The main advantage of this framework relative to our simple two-period framework is that wealth is now endogenous and we can compute an invariant wealth distribution, and, perhaps more importantly, that we can use it to understand the reaction of constrained and unconstrained dynasties to transitory and permanent income shocks over multiple periods. The drawback

of this more complex framework is that our analysis is mostly based on numerical simulations only. As a result of an idiosyncratic unexpected temporary income shock, unconstrained individuals first run down their financial assets until they are at the borrowing constraint. Once there, they start borrowing using the location asset and so downgrade their location in order to minimize fluctuations in their level of consumption. This downgrading of location continues until individuals reach the worse location they are willing to go to, or the income shock reverts to the high value. Once the temporary shock has reverted, individuals go back to the initial location progressively.³

The implications of the ‘location as an asset’ view are sharp. Negative (positive) front-loaded shocks should make constrained individuals downgrade (upgrade) location, while unconstrained agents should not change their location. To contrast these predictions with empirical evidence we use a detailed individual panel data from France. The data covers the universe of workers and provides us with the wage of employed individuals together with their location and a number of other characteristics. We use these data to study the location decisions of individuals that experience unemployment spells. We rank locations according to their average income and see how the rank of an individual’s location changes when they find a new job. The results are stark, individuals that start at the bottom quintile of the distribution of income in their location downgrade their location by about 5 percentile points relative to individuals at the top quintile. Conditional on moving, the location rank falls by as much as 18 percentile points. These results are robust to a battery of municipality, occupation, industry, birthplace and age fixed effect. Of course, the relative downgrading of an individual’s location might potentially be the result of obtaining a job that pays lower wages after their unemployment spells and this might be related to their origin wage percentile. However, when we control for wage growth and the interaction of wage growth and the original wage percentile the results are, if anything, larger. This indicates that the downgrading of location is not simply the response to lower wages. It also shows that, at least partly, location choice determines wage growth. Consistently, the effect of origin wage percentile declines with wage growth, suggesting that the trade-off between location and future income is significant in the data, particularly for constrained individuals. This is exactly the effect emphasized by the location as an asset view. These facts are of independent interest and are, as far we know, unknown to the literature. We can estimate them precisely given the large number of observations in our data.

There is a large literature documenting the large variation in income levels and other outcomes across locations.⁴ Kennan and Walker (2011) argue forcefully that inter-state migration decisions are made based

³Our infinite horizon model shares many features with dynamic portfolio problems with investors who face a credit constraint on risk-free bonds. Thus, we build a Huggett (1993) economy with a second asset: the ‘location asset’. In particular, our model could be viewed as one in which possibly constrained entrepreneurs choose in which project to invest (the location), subject to a collateral constraint. Related work includes but is not limited to Angeletos (2007) and Moll (2014). Our framework is distinct from those in two dimensions. First, we model both risk-aversion and idiosyncratic additive income shocks on the investor side, leading individuals to use the location asset to smooth consumption when they are close to the constraint. Second, individuals in our model always wish to hold a convex combination of both assets, due to the endogenously nonlinear returns of the ‘location asset’.

⁴See Wilson (1987), Denton and Massey (1993), Cutler and Glaeser (1997), Desmet and Rossi-Hansberg (2013), Altonji and Mansfield (2014), and Hsieh and Moretti (2018) among many other.

on income prospects, but are also influenced importantly by geographic differences. In fact, Diamond (2016) and Giannone (2017) show that the U.S. has experienced increasing skill segregation, indicating that spatial gaps are not diminishing. Kaplan and Schulhofer-Whol (2017) show mobility in the U.S. is declining.⁵ Most equilibrium analysis of individual location choices is either cast in partial equilibrium and so does not consider the valuation side of the ‘location as an asset’ view (like Kennan and Walker, 2011, or Diamond, 2016) or static and based on a simple spatial equilibrium condition that does not include the investment aspect of location decisions (like Desmet and Rossi-Hansberg, 2013, Allen and Arkolakis, 2014, Redding, 2016, or Caliendo et al., 2018). Giannone (2017) and Desmet, Nagy and Rossi-Hansberg (2018) do provide dynamic general equilibrium setups with costly migration, but migration decisions only provide static gains or losses. In Caliendo, Dvorkin and Parro (2017) agents solve forward looking problems in deciding their location but they simply consume their income and so do not solve a consumption-savings decision or accumulate wealth.

The view of investment as an asset was hinted at initially by Sjaastad (1962). Lucas (2004), Morten (2017) and Calvacanti-Ferreira et al. (2016) also present evidence and arguments to view migration as a stepping-stone or a form of self-insurance.⁶ Some of the most detailed studies of mobility for low income, and likely constrained individuals, are consistent with the ‘location as an asset’ view. For example, in the “Moving to Opportunity” randomized experiment, conditioning aid on upgrading location reduced the use of housing vouchers by about a third (21 percentage points). Furthermore, while the literature using this experiment initially found that economic outcomes were not affected by an upgrade in location (Duncan et al., 2013), the most recent studies have found strong evidence that the outcomes for children that moved when young are positive (Chetty, et al., 2016, and Davis et al., 2017), consistent with our emphasis on the investment dimension of location decisions rather than on the current benefits. Using tax records, Chetty and Hendren (2017) found a trade-off between child future earnings and rents. They estimate that a 1% increase in a child’s future earnings can be achieved by moving to a location with a median rent that is \$176 higher. The location as an asset view argues that constrained agents might not want to take what seems like a good bargain, since they are constrained and want to borrow not invest further.

The rest of the paper is organized as follows. The next section, Section 2, introduces the simple model necessary to make precise our notion of location decisions as investment decisions. This simple two period model is then extended to an infinite horizon model in Section 3. In that section we present examples of the implied dynamic consumption, asset, and location paths of individuals. Section 4 presents our empirical analysis using the French individual level panel to show that agent’s location decision respond to non-employment spells as our theory predicts. Section 5 concludes. An Appendix includes the technical proofs, additional robustness tests, and detailed data descriptions.

⁵Kaplan and Schulhofer-Wohl (2017) link the decline in U.S. mobility to falling wage differentials within occupations.

⁶Fernandez and Rogerson (1998) and Fogli and Guerreri (2017) discuss the trade-off between location and children education.

2 A Simple Model

We aim to provide the simplest setup in which our ‘location as an asset’ view can be made precise. Because we need location to be an investment, we need a model with at least two periods. Hence, we model an economy over periods 0 and 1. The economy consists of a unit mass of individuals that differ in their skill, $s \in [\underline{s}, \bar{s}]$, and their income in period 0 and 1, $\{y_t\}_{t=0}^1 \in [\underline{y}_t, \bar{y}_t]$. The income of the individual in period 0 includes her labor income plus any wealth she is initially endowed with. In sum, an individual is characterized by a triplet (y_0, y_1, s) . We denote the joint probability density function over these outcomes by f and the cumulative distribution by F .

There is a continuum of locations or ‘cities’. We classified cities according to the complementarity of the returns from living in them with the skills of individuals. We denote locations by an index $z \in [\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$. The density of cities with characteristic z is given by h with cumulative density H . The skill of an individual determines the benefits from locating in cities. We assume that the returns for an individual of skill s to living in city z are given by zs . Agents can move freely across locations. Hence, the supermodularity of this function will lead to positive assortative matching conditional on other individual characteristics, as we describe below.

The population density, $L(z)$, of individuals living in cities of type z , as well as land rents, $q(z)$, in those cities are determined endogenously. We assume that the cost of supplying housing increases with population size due to some form of decreasing returns. Hence,

$$q(z) = Q(L(z)) \text{ for } z \in [\underline{z}, \bar{z}]$$

where $Q(0) = 0$ and Q strictly increasing. That is, housing is free in locations without population and rents are strictly increasing in city size.

Individuals have access to a risk free bond with gross interest $R > 1$. We assume that this world interest rate is exogenous and determined in world markets.⁷ Agents are subject to an ad-hoc borrowing constraints that limits their asset holdings between period 0 and 1, a , to be above some level \underline{a} . Hence, if, for example $\underline{a} = 0$, agents can only save but not borrow with the financial asset.

2.1 Asset and Location Choices

Households maximize lifetime utility with a discount factor given by $\beta \leq 1$. For simplicity we specify the period utility function as $u(c) = \log c$ but virtually all our results should go through for any concave utility function that satisfies Inada conditions. The problem of a household is then to choose consumption in each

⁷Technically, we only need $R > 0$, which we can allow without loss of generality. In addition, it would be simple to endogenize the interest rate R through an asset market clearing condition without changing any of our core results.

period, purchases of the risk free bond, and location in period 1, to solve

$$\begin{aligned}
 V(y_0, y_1, s) &= \max_{c_0, c_1, a, z} \log c_0 + \beta \log c_1 & (1) \\
 \text{s.t. } c_0 + a + q(z) &= y_0, \\
 c_1 &= zs + y_1 + Ra, \\
 a &\geq \underline{a}.
 \end{aligned}$$

That is, individuals maximize utility subject to budget constraints each period, as well as the borrowing constraint. In period zero, an agent's income includes anything he earns today and all of his wealth. Note that we have abstracted from any returns from the complementarity between an agent's skill and the city where she starts (say, z_0s). We think of this term as also being embedded in y_0 . Not explicitly recognizing this term explicitly avoids carrying z_0 as a state variable in the consumer problem. This is without loss of generality given that free mobility implies that current location only affects an agent's decisions through current income.

Note also that we make the agent pay rent one period in advance. So land rent for their chosen z location, $q(z)$, enters the left-hand-side of the period 0 budget constraint only. Rent paid for living in location z_0 in period 0 is not modeled and would simply be included in the resulting period 0 income. Making household pay rent one period in advance underscores the investment nature of the location choice. Namely, it recognizes that the good jobs, amenities, or education associated with living in a good location are enjoyed over time and not necessarily immediately after arriving there.

The problem in (1) also abstracts from income risk. In the next section we write a multi-period extension with uncertainty about the realization of the income process. However, in this simple model without uncertainty, the location asset is used to transfer consumption across time, but not across states of nature or for precautionary purposes. Of course, in a richer environment the location asset could also be used for these alternative purposes.

The first-order conditions of the problem in (1) imply the standard 'Financial Euler equation'

$$\frac{c_1^*(y_0, y_1, s)}{\beta c_0^*(y_0, y_1, s)} \geq R \text{ for all } (y_0, y_1, s), \quad (2)$$

with equality if and only if the borrowing constraint is not binding, namely $a^*(y_0, y_1, s) > \underline{a}$. We denote all individual optimal choices with an asterisk (*).

Absent borrowing constraints, the desired asset holdings of an individual (y_0, y_1, s) , denoted by $\tilde{a}(y_0, y_1, s)$, are given by their income net of rents in period zero ($y_0 - q(z)$) minus permanent consumption, which is

given by $\left(y_0 + \frac{y_1 + z^* s}{R} - q(z^*)\right) / (1 + \beta)$.⁸ Namely,

$$\tilde{a}(y_0, y_1, s) = y_0 - q(z^*(y_0, y_1, s)) - \frac{y_0 + \frac{y_1 + z^*(y_0, y_1, s)s}{R} - q(z^*(y_0, y_1, s))}{1 + \beta}.$$

Thus, actual savings in the financial asset are given by

$$a^*(y_0, y_1, s) = \max\{\tilde{a}(y_0, y_1, s), \underline{a}\}.$$

Free mobility implies that individuals are never constrained in the ‘location asset’. Hence, for all agents, the location decision yields a ‘Migration Euler equation’ given by

$$\frac{c_1^*(y_0, y_1, s)}{\beta c_0^*(y_0, y_1, s)} = \frac{s}{q'(z^*(y_0, y_1, s))}, \quad (3)$$

for all (y_0, y_1, s) .

Hence agents can optimize their intertemporal consumption path by choosing their holdings of financial assets and what we have dubbed the ‘location asset’. To make the analogy with a standard asset more precise, we can propose two interpretations. First, one in which each location z constitutes an asset, and agents moving to location z buy the asset, and the ones moving out sell it. How much of it they buy is limited by their housing demand and labor supply. Here, for simplicity, we have limited labor supply and housing demand to be equal to one. The return of the asset depends on the skill of the individual, s , and is given by the right-hand-side of equation (3), namely, $s/q'(z^*)$.

An alternative interpretation is to consider only a single asset with unit cost. The quantity purchased of the asset is equal to the housing costs, q , and returns of the asset depend both on the quantity purchased and the skill of the individual. Again, those returns are given by $s/q'(z^*)$. Under both these interpretations, the individual’s problem (1) can be seen as a standard portfolio choice problem in which the risk-free bond is subject to a borrowing constraint, and the return to the ‘location asset’ is endogenously nonlinear and specific the individual’s skill.

We are ready to define a competitive equilibrium in our economy.

Definition 1 *Given a distribution F of triplets $(y_0, y_1, s) \in [\underline{y}_0, \bar{y}_0] \times [\underline{y}_1, \bar{y}_1] \times [\underline{s}, \bar{s}]$ and an interest rate R , an equilibrium is a set of individual decision functions $c_0^*, c_1^*, a^* : [\underline{y}_0, \bar{y}_0] \times [\underline{y}_1, \bar{y}_1] \times [\underline{s}, \bar{s}] \rightarrow \mathbb{R}_+$ and $z^* : [\underline{y}_0, \bar{y}_0] \times [\underline{y}_1, \bar{y}_1] \times [\underline{s}, \bar{s}] \rightarrow [\underline{z}, \bar{z}]$, and rent and population functions $q, L : [\underline{z}, \bar{z}] \rightarrow \mathbb{R}_+$ such that*

- *individuals solve the problem in (1) and*

⁸Whenever it is clear by the context we abbreviate optimal choices and do not write the dependence on the agent’s type. Namely, we might write z^* instead of $z^*(y_0, y_1, s)$.

- land rents are such that $q(z) = Q(L(z))$ for $z \in [\underline{z}, \bar{z}]$ where city population $L(z)$ satisfies

$$\int_{\underline{z}}^z L(z) H(dz) = \int_{\underline{y}_0}^{\bar{y}_0} \int_{\underline{y}_1}^{\bar{y}_1} \int_{\underline{s}}^{\bar{s}} \mathbf{1}[z^*(y_0, y_1, s) \leq z] F(dy_0, dy_1, ds) \text{ for all } z \in [\underline{z}, \bar{z}] \quad (4)$$

and $\mathbf{1}$ denotes the indicator function.

Condition (4) guarantees that the number of people in locations worse than z (the left-hand-side of the condition) is equal to the number of people that choose to live in those locations (the right-hand-side of the condition). Note that Condition (4) has to hold for all $z \in [\underline{z}, \bar{z}]$ and so it implicitly determines the population density function $L(z)$.

2.2 Equilibrium Allocation and House Rents

In order to understand agent's location choices, consider a city z in which an unconstrained individual (y_0, y_1, s) lives. Because $a^*(y_0, y_1, s) > \underline{a}$, equation (2) holds with equality and so the returns she faces on the financial and the location asset need to be equal. That is,

$$R = \frac{s}{q'(z^*(y_0, y_1, s))}.$$

This implies that unconstrained individuals sort into cities on the basis of their skill component s only. Then, if $q(\cdot)$ is a strictly increasing function (something we show below), there exists a matching function $\mathcal{Z}^U(s) = z^*(y_0, y_1, s)$ for unconstrained individuals, such that

$$R = \frac{s}{q'(\mathcal{Z}^U(s))}.$$

Furthermore, when $q(\cdot)$ is convex (which we also show below), $\mathcal{Z}^U(s)$ is strictly increasing. Of course, whether individuals are constrained on the financial asset depends on their income path and skill, and the resulting location choice. For example, a flat income path with y_0 high relative to the values of future income, y_1 , and skill, s , implies that the individual is not constrained.

Now consider an individual with the same y_1 and s but low enough $y'_0 < y_0$ such that she is constrained. This individual has a larger marginal rate of substitution than the interest rate, so the Financial Euler equation (2) holds with strict inequality. Since the agent can still use the location asset, and so (3) holds, this implies that $s/q'(\mathcal{Z}^U(s)) = R < s/q'(\mathcal{Z}^C(y'_0, y_1, s))$ where $\mathcal{Z}^C(y'_0, y_1, s)$ is the constrained agent's location choice. Note that the constrained agent's location choice depends on all the individual characteristics, not just s . Hence, for $q(\cdot)$ strictly increasing, $\mathcal{Z}^U(s) > \mathcal{Z}^C(y'_0, y_1, s)$. Constrained individuals locate in cities with lower land rents and lower returns to skill than unconstrained individuals with the same skills. The reason is that they use the location asset rather than the financial asset to adjust their intertemporal consumption path. More specifically, they borrow using the location asset to transfer resources to the present, something

financial markets do not allow them to do.

$\mathcal{Z}^C(y_0, y_1, s)$ is increasing in y_0 and in fact will converge to $\mathcal{Z}^U(s)$ as we increase y_0 . In contrast, it is decreasing in y_1 , since larger future income results in larger need to borrow from the future and therefore more use of the location asset to do so. Finally, more skilled individuals locate in better cities, whether constrained or unconstrained, due to the skill complementary we introduce in individual earnings. Note that the reason the individual location choice is always uniquely determined in our setup is the supermodular income in z and s . In contrast, if agents had identical skills, they would be indifferent about where to locate when unconstrained, but their use of the location asset to transfer consumption to the present would still determine their location choice when constrained. We formalize this discussion in the following lemma that characterizes the location decision of agents.

Lemma 1 *There exists a pair of matching functions $\mathcal{Z}^U(s)$ and $\mathcal{Z}^C(y_0, y_1, s)$ such that individual (y_0, y_1, s) chooses city*

- $z^*(y_0, y_1, s) = \mathcal{Z}^U(s)$ if $y_0 \geq Y_0(y_1, s)$, so she is unconstrained, and
- $z^*(y_0, y_1, s) = \mathcal{Z}^C(y_0, y_1, s) < \mathcal{Z}^U(s)$ if $y_0 < Y_0(y_1, s)$, so she is constrained,

where

$$\begin{aligned} Y_0(y_1, s) &= \{y_0 \mid a^*(y_0, y_1, s) > \underline{a}\} \\ &= (1 + \beta^{-1})\underline{a} + q(\mathcal{Z}^U(s)) + \frac{y_1 + s\mathcal{Z}^U(s)}{\beta R} \end{aligned}$$

and \mathcal{Z}^U and \mathcal{Z}^C are determined by a system of ordinary differential equations described in Appendix A.1.

Proof. See Appendix A.1. ■

Lemma 1 characterizes the threshold for current income y_0 that determines whether an individual is constrained using the function $Y_0(y_1, s)$. Because the rent function is increasing in z as we show below, and since $\mathcal{Z}^U(s)$ is increasing in s , this threshold is increasing in both arguments. More future income makes unconstrained individuals want to consume more in the present and therefore makes the constraint on borrowing more binding. Similarly, more skilled individuals will earn more in the future and will live in more expensive cities, making the constraint more binding.

Of course, given the monotonicity of $\mathcal{Z}^U(s)$ and $\mathcal{Z}^C(y_0, y_1, s)$ in s , we can define the inverse as $\mathcal{S}^U(z) = \mathcal{Z}^{U-1}(z)$ and $\mathcal{S}^C(y_0, y_1, z) = \mathcal{Z}^{C-1}(y_0, y_1, z)$. These functions then tell us the skill of the set of constrained and unconstrained individuals that live in a given city z . In equilibrium, unconstrained individuals always locate in better cities than constrained ones, hence there exists a threshold \hat{z} such that for $z < \hat{z}$ all individuals in the city are constrained and above that we have a mixed of constrained and unconstrained

individuals. The best city, \bar{z} , is an exception and has no constrained agents. The following corollary states these results formally.

Corollary 2 *There exists a threshold \hat{z} such that individuals in city $z \geq \hat{z}$ are either*

- *unconstrained with skill $s = \mathcal{S}^U(z)$ and $y_0 \geq Y_0(y_1, \mathcal{S}^U(z))$, or*
- *constrained with $s = \mathcal{S}^C(y_0, y_1, z) > \mathcal{S}^U(z)$, and*

$$Y_0(y_1, \mathcal{S}^U(z)) > y_0 \tag{5}$$

$$\mathcal{S}^C(y_0, y_1, z) = \frac{\mathcal{S}^U(z)(y_1 + R\underline{a})}{\beta R(y_0 - \underline{a} - q(z)) - z\mathcal{S}^U(z)} \tag{6}$$

In cities $z < \hat{z}$, all individuals are constrained, and $\mathcal{S}^C(y_0, y_1, z) = \frac{q'(z)(y_1 + R\underline{a})}{\beta(y_0 - \underline{a} - q(z)) - zq'(z)}$.

Proof. Direct corollary of Lemma 1. ■

Figure 1 represents these results graphically. We have discussed all the elements in the figure except for \tilde{z} that represents the lowest city that has non-negative housing rents. Namely, \tilde{z} is implicitly defined by $q(\tilde{z}) = 0$. If $q(z)$ is strictly increasing in z , any city with $z < \tilde{z}$ is not feasible. Note that the upper bound of the correspondence of skills that live in the city is given by $\mathcal{S}^C(y_0, y_1, z)$ evaluated at the lowest current income (denoted by \underline{y}_0) and highest future income (\overline{y}_1). Namely, the most constrained individual in the city, which is the highest skilled individual using the location asset the most. Note that below \hat{z} the city has only constrained individuals, and only the lowest skilled individuals locate in the worst city \tilde{z} (as long as \underline{z} is low enough).

We can also represent graphically the set of current income levels, y_0 , of individuals that locate in a given city. Of course, current income and initial wealth are indistinguishable in our two-period setup. We do so in Figure 2. In city z , all individuals with incomes $y_0 \geq Y_0(y_1, \mathcal{S}^U(z))$ are unconstrained and locate according to their skill level only. Other individuals that locate in those cities are constrained and have low income, and either high skills, high future income or both. Because lower current income leads individuals to choose worse cities, it must be that the lowest income present in a given city z is the income of the individual with the highest incentives to save in the location asset. Namely, the highest skill agent with the lowest future income present in the city. This lower bound, denoted by $\underline{Y}_0(z)$ in the figure, can be found by evaluating the expression for \mathcal{S}^C in equation 6 in Corollary 2 at \bar{s} and the lowest future income \underline{y}_1 .

We finish the discussion of an equilibrium in our simple two-period economy with a characterization of the house rent schedule. As we alluded already above, land rents are increasing in z since higher z cities yield higher income for individuals of all skills. Furthermore, the complementarity between z and s implies that the highest skilled unconstrained individuals locate there, which implies that rents grow more than proportionally with city type, as does the income of its unconstrained residents. Hence, rents are convex. Figure 3 illustrates such a rent function.

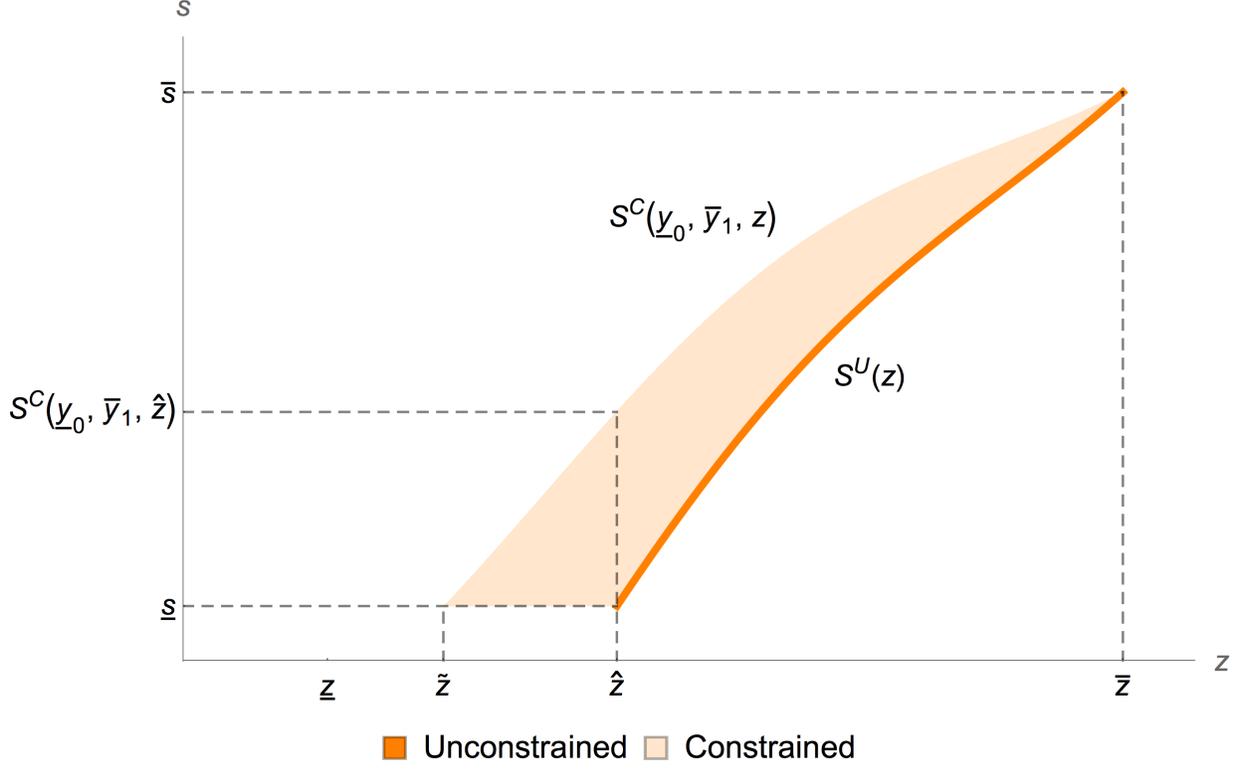


Figure 1: Allocation of Skills to Cities

In cities with unconstrained individuals the slope of the rent function is given by the $\mathcal{S}^U(z)/R$. Namely, the slope of the rent function is determined by the skill of unconstrained individuals in the city and is inversely proportional to the interest rate. Thus, a low interest rate implies that the house rent schedule is steeper. Since the return of the location asset for an unconstrained individual with skill s is $s/q'(Z^U(s))$, this naturally also implies a lower return of the competing location asset by no-arbitrage. That is, lower returns in the financial market result in steeper rents that reduce the return of the location asset. Furthermore, a lower interest rate R implies that more agents wish to borrow and hence are constrained. This implies more downgrading and segregation. So the model predicts that periods of low interest rates should be periods of increasing rent differentials across cities and more segregation, reminiscent of the pre and post- 2008 crisis housing markets around the world. We formalize these results in the following lemma.

Lemma 3 *The equilibrium house rent function has the following properties:*

- $q(z)$ is increasing and convex,
- for $z \geq \hat{z}$, $q'(z) = \frac{\mathcal{S}^U(z)}{R}$, and
- $\frac{\partial q'(z)}{\partial R} < 0$ for $z \geq \hat{z}$ if $\bar{s} - \underline{s}$ is sufficiently small.

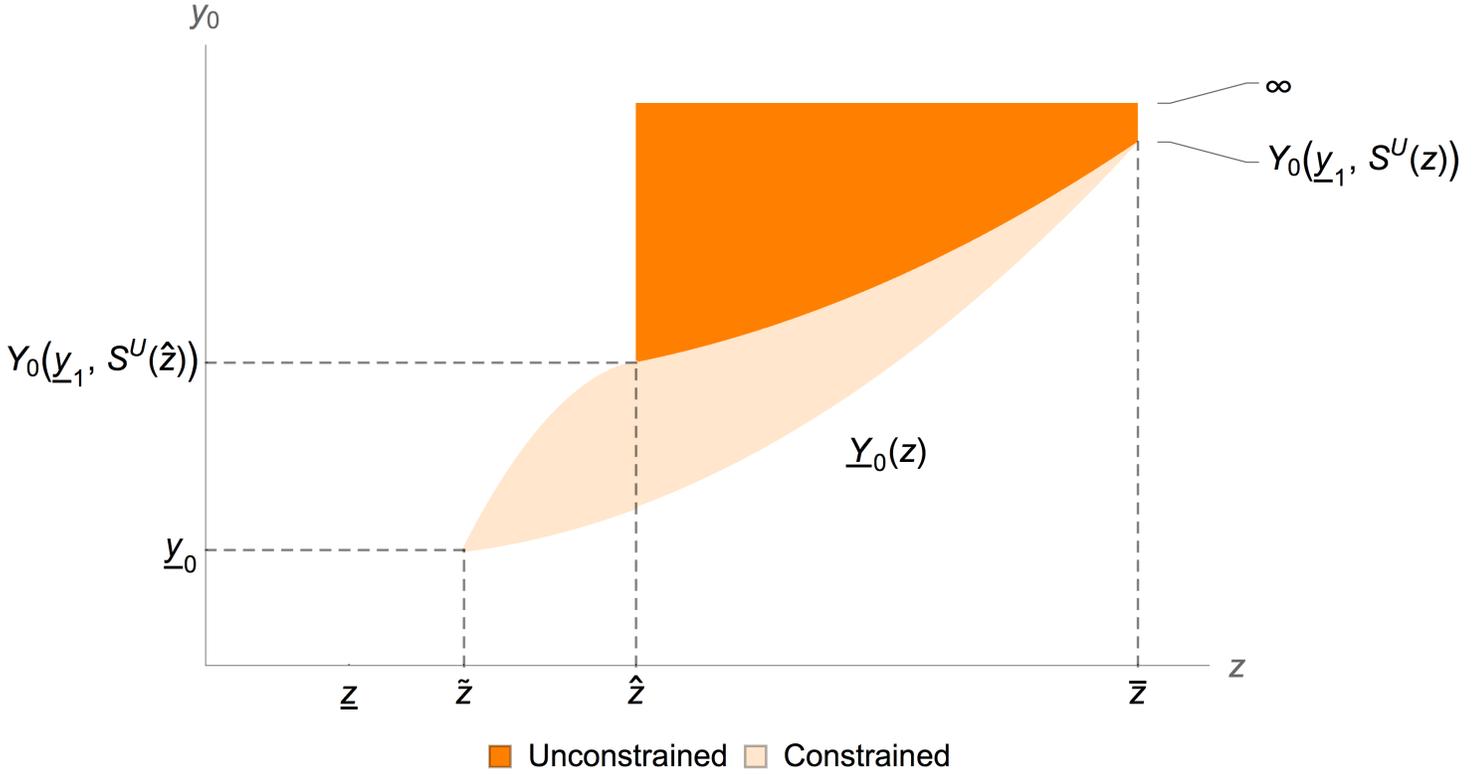


Figure 2: Allocation of Income Groups to Cities

Proof. See Appendix A.2. ■

2.3 Optimal Allocation

The equilibrium allocation of the model described above is inefficient due to the presence of borrowing constraints. The inefficiency is reflected in the use of the location asset by constrained individuals. Their use of the location asset ameliorates the effect of the financial constraint. However, because it reduces total output in the second period by driving agents to locations where they earn less, the resulting allocation is still inefficient relative to an economy without financial constraints.

Finding an efficient allocation can be broken down in two parts. First, the problem of allocating individuals across locations to maximize discounted second period output net of housing costs, and second the allocation of consumption in both periods across individuals of different types. We focus on the solution of the first part of the planner's problem. The second part is a redistribution problem that depends on the chosen social welfare function and for any standard welfare function the solution is increasing in the total output generated by the allocation of agents across locations.

Given the assumed supermodularity between s and z , the planner allocation necessarily involves a one-

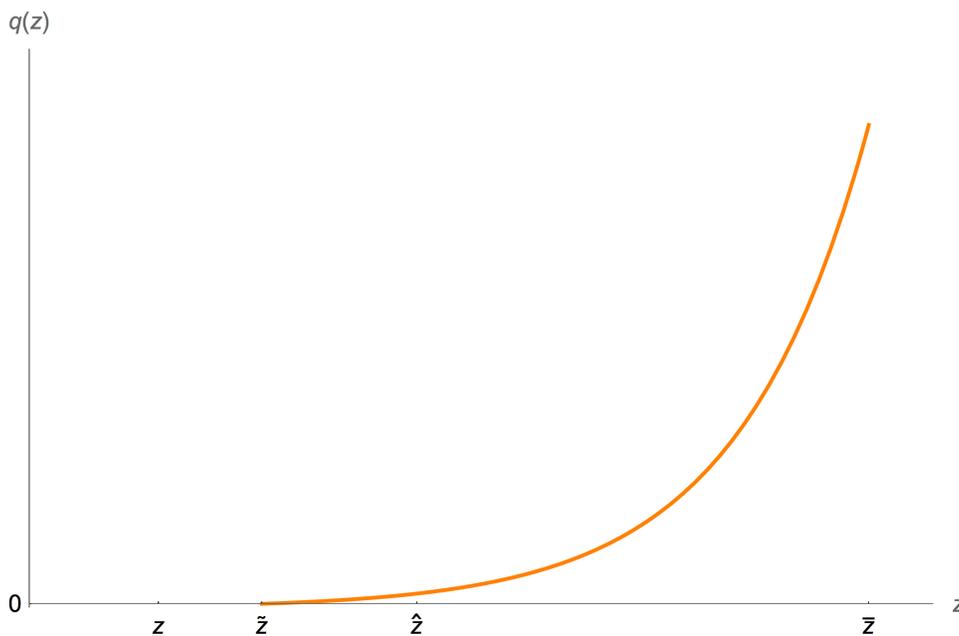


Figure 3: House Rents Across Cities

to-one increasing matching function. Namely, the solution exhibits positive assortative matching. Hence, in contrast to the equilibrium allocation, only one type of agent locates in a given city. We show this rigorously in the following lemma.

Lemma 4 *Consider the problem of a planner in a small open economy that does not face credit constraints and has access to an asset with exogenous return R . Then:*

- *the planner allocates individuals according to an increasing matching function $Z^{SP}(s)$, and*
- *the decentralized allocation yield strictly less (i) present value of output, and (ii) present value of output net of housing costs*

Proof. See Appendix A.3. ■

2.4 Placed-Based Policies

The equilibrium described above determines the distribution of population across cities, $L(z)$, for all $z \in [\underline{z}, \bar{z}]$ with $L(z) > 0$ for $z \in [\tilde{z}, \bar{z}]$. In the equilibrium allocation, agents with low values of s that are constrained decide to locate in the lower range of cities because they use the location asset to borrow. We now want to consider the effect that place based-policies might have on welfare for the different types of agents. Place-based policies aim to improve the characteristics of some of the worse locations in the economy. This is

naturally costly, and implies taxing other locations. Therefore, as a stylized representation of these policies, consider policies that shrink the range of characteristics of equilibrium cities $[\underline{z}, \bar{z}]$ to a singleton $\{z_0\}$, keeping the mass of cities constant. We choose z_0 to guarantee that the average income that individuals derive from cities stays unchanged, namely, $E[sz] = z_0 E[s]$.⁹ Thus, this policy captures the essential elements of place-based policies if they are implemented without generating any aggregate loss of resources. Note also that positive sorting between skills and city types implies that

$$z_0 = E \left[z \times \frac{s}{E[s]} \right] > E[z].$$

That is, the targeted city type is better than the average.

To explain our general result below it is useful to start with an example where $\underline{s} = 0$. Namely, the lowest skilled individuals in the economy have zero skill and, therefore, derive no benefits from living in better cities. These individuals in equilibrium locate in the worst cities in the economy, \underline{z} , and pay zero rent $q(\underline{z}) = 0$. Naturally, such individuals will be worse off if we implement our place-based policy. In the equilibrium with place-based policies rents are positive and identical in all cities, but for the lowest skilled individuals the benefits of locating in the improved cities are still zero. Hence, anyone with $\underline{s} = 0$ necessary losses from the policy. By continuity, there is a range of individuals with $s > 0$ that are also worse off after the policy. If they have some skills, they benefit in terms of future income, but the increase in rents still dominates. Or, in other words, the policy prevents them from borrowing with the location asset. Something they would like to do.

As long as $\underline{s} = 0$, the logic above applies for any policy that reduces the range of cities at the bottom of the distribution. Namely, any policy that improves the worst city that agents have access to (and therefore increases its equilibrium rents). Of course, this logic also relies on keeping the mass of cities constant. This is intuitive, place-based policies that improve the worse cities in the equilibrium allocation but that allow for the creation of new low- z cities would achieve little.

The logic described above for the case of $\underline{s} = 0$ can be extended to a more general setting with $\underline{s} > 0$, when $Q(L(z)) = L^\eta$ with $\eta < 1$.¹⁰ In this case we can characterize the set of individuals that lose using the matching functions. The individuals that are guaranteed to lose are the ones between the lowest skill, and the skill of the individuals that locate, in the original equilibrium, in the average city. The reason is, again, that up to that point the convexity of housing prices implies that the increase in rents associated with the policy does not compensate the future gain in income for these agents. That is, these agents get low returns for the location asset, so they like to use it to borrow, not to save. This is particularly true for constrained individuals, so the set of skills of constrained individuals that lose is larger than the set of unconstrained

⁹The expectation on the left-hand-side is taken with respect to the equilibrium allocation in space in the competitive equilibrium with different cities, before the policy change. The expectation on the right-hand-side involves only the exogenous marginal distribution of skill.

¹⁰This is a natural assumption that holds, for example, in the standard circular monocentric city model with a central business district and commuting (as in Desmet and Rossi-Hansberg, 2013). In that case, $\eta = 1/2$.

individuals that lose from the policy. The next lemma presents the formal result.

Lemma 5 *Suppose house rents are concave in population, i.e. $Q(L(z)) = L^\eta$ with $\eta < 1$. Then a place-based policy that makes all cities have characteristic z_0 makes*

- *all unconstrained agents with $s \in [\underline{s}, S^U(E[z])]$ worse off and*
- *all constrained agents (y_0, y_1, s) with $s \in [\underline{s}, S^C(y_0, y_1, E[z])]$ worse off.*

Since $S^C(y_0, y_1, E[z]) > S^U(E[z])$, the set of skills of constrained individuals that are worse off is larger.

Proof. See Appendix A.4. ■

2.5 The Location Effect of Front and Back-Loaded Shocks

The results above can also be used to describe how agents react to shocks of different types. We are particularly interested in income shocks that affect the relative slope of an individual's income path. Namely, shocks that affect income today, y_0 , relative to income tomorrow, $y_1 + sz$. These shocks will induce agents to adjust their savings using the financial and location assets. In Section 4 we study how workers in France reallocate across regions as a result of an unemployment spell. An unemployment spell is a front-loaded shock for individuals that receive little or no severance pay, since it reduces income today relative to income tomorrow. So we can contrast the model's predictions with our observations for France. Other front-loaded shocks include declines in the compensation of particular occupations or particular industries. The shock is front-loaded because individuals and their descendants can adjust their future occupation and industry, but are stuck in the short run.

Consider an individual (y_0, y_1, s) that experiences an idiosyncratic negative front-loaded shock that decreases y_0 to $y'_0 < y_0$ but increases y_1 to $y'_1 \geq y_1$. Because the shock is idiosyncratic, it does not affect the equilibrium matching function or rent schedule. The results in Lemma 1 imply that agents that are constrained will use the location asset more and will downgrade their location, since $\mathcal{Z}^C(y'_0, y'_1, s) < \mathcal{Z}^C(y_0, y_1, s)$. Unconstrained individuals that become constrained due to the shock also downgrade their location, since $\mathcal{Z}^C(y'_0, y'_1, s) < \mathcal{Z}^U(s)$. In contrast, unconstrained individuals that remain unconstrained (individuals such that $y_0 > y'_0 > Y_0(y'_1, s) \geq Y_0(y_1, s)$) stay where they are, since $\mathcal{Z}^U(s)$ is independent of the income path. Hence, constrained individuals, or those that become constrained, borrow more using the location asset, while unconstrained individuals use the financial asset to transfer consumption to the present. Of course, since what matters for the argument is the slope of the income path, a positive back-loaded shock has a similar effect on location choices and the use of the location asset.

A positive front-loaded shock or a negative back-loaded shock has exactly the reverse effect. Constrained individuals, or individuals that become unconstrained, save with the location asset and upgrade location. Individuals that were, and remain, unconstrained use the financial market to save and do not change their use of the location asset.

Note that permanent adverse (or positive) shocks can imply a change in the slope of the income profile. For example a permanent adverse shock that increases both y_0 and y_1 induces borrowing if $y'_0 - y'_1/\beta R < y_0 - y_1/\beta R$. Such a shock then generates the same qualitative effects on the use of the location asset as a front-loaded negative shock. In contrast, if $y'_0 - y'_1/\beta R > y_0 - y_1/\beta R$, the shock induces extra savings and so has a similar qualitative effect than a back-loaded negative shock. Of course, if $y'_0 - y'_1/\beta R = y_0 - y_1/\beta R$, locations remain unchanged.

As a last possibility consider an individual that acquires more skill, namely, an increase in s . Because s increases income in the future, some of the implications of the increase in s are similar to those of a back-loaded positive shock. On top of this, an increase in s increases the return of the location asset relative to the financial asset which implies that agents want to save more using the location asset. Hence, they want to upgrade their city. Lemma 1 tells us that the the second effect always dominates, given that both $\mathcal{Z}^C(y_0, y_1, s)$ and $\mathcal{Z}^U(s)$ are increasing in s .

In the context of our simple model and the results described above, we can think of a worker losing her job as changing current income from y_0 to y'_0 . Once the worker find a new job next period she again earns y_1 . If the worker receives unemployment benefits that are, say, a fraction $\kappa < 1$ of her last salary, then $y'_0 = y_0\kappa$ and the shock is a front-loaded negative shock that makes individuals downgrade if constrained and not relocate if unconstrained. If in contrast the worker receives, say, a severance pay that makes the current payment larger than when employed, $y'_0 = \sigma(y_0)y_0 > y_0$ (perhaps at the cost of a lower y_1 when she finds a job), then the shock is a front-loaded positive shock that will make constrained individuals upgrade. Now, if the generosity of severance pay depends on the level of income (e.g. $\sigma(y_0)$ is increasing), then high income individuals that are constrained upgrade location when they lose their job while low income ones downgrade. Overall, however, since high income individuals are less constrained than low income ones, we expect the use of the location asset to be more pronounced among people in the latter group. We contrast this exact implications with French data in Section 4.

3 Infinite Horizon Model

In this section we extend our model to an infinite horizon economy. The key differences with the model presented in the previous section, is that now agents live forever and receive an idiosyncratic income stream y_t . Depending on their skill, location, asset holdings, and income, they make consumption and savings decisions. To do so they use the financial market subject to a borrowing constraint and the location asset by choosing where to live. As before, cities differ in their return to skill and their rent. Also as before, one can view individuals as solving a two-asset portfolio choice problem subject to a borrowing constraint on the risk-free bond. In contrast to the two period model, the infinite horizon version determines the invariant distribution of wealth in the population and therefore the wealth composition of cities as well.

In any period t , infinitely-lived individuals receive an idiosyncratic income shock y_t , which follows a first-order Markov chain with states y_1, \dots, y_N and a given transition matrix, Λ . Throughout we assume

that individuals have a permanent skill s .¹¹ In period t , an individual in location z_t with an asset level a_t , chooses how much to consume c_t , how much to save a_{t+1} in a one period risk-free bond with interest rate R , and where to live next period, z_{t+1} . Agents can move freely across locations. Their income in period t is $y_t + sz_t$. To go to location z_{t+1} , they need to pay the rent $q(z_{t+1})$ one period in advance, i.e. in period t . Finally, we assume that the risk-free bond is subject to an ad-hoc credit constraint $a_{t+1} \geq \underline{a}$.

Given an increasing and concave flow utility function u satisfying Inada conditions, and a discount factor $\beta < 1$, individuals maximize

$$V(a_t, z_t, y_t, s) = \max_{\{a_{t+1}, z_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (7)$$

$$s.t. \quad c_t + a_{t+1} + q(z_{t+1}) = y_t + sz_t + Ra_t,$$

$$a_{t+1} \geq \underline{a}.$$

If we denote optimal choices with an asterisk, as in the two period model the solution to this dynamic optimization problem yields a financial Euler equation

$$\frac{u'(c_t^*(a_t, z_t, y_t, s))}{\beta E_t[u'(c_{t+1}^*(a_{t+1}, z_{t+1}, y_{t+1}, s))]} \geq R$$

that holds with equality if and only if $a_{t+1}^*(a_t, z_t, y_t, s) > \underline{a}$. Also similarly, free mobility implies a migration Euler equation given by

$$\frac{u'(c_t^*(a_t, z_t, y_t, s))}{\beta E_t[u'(c_{t+1}^*(a_{t+1}, z_{t+1}, y_{t+1}, s))]} = \frac{s}{q'(z_{t+1}^*(a_t, z_t, y_t, s))}$$

which implies that

$$\frac{s}{q'(z_{t+1}^*(a_t, z_t, y_t, s))} \geq R \quad (8)$$

with equality if and only if $a_{t+1}^*(a_t, z_t, y_t, s) > \underline{a}$. Note that, again we have that, for non-constrained individuals, city choice $z_{t+1}^*(a_t, z_t, y_t, s)$ only depends on skill s .

Denote by F_t the joint distribution of the four-tuple (a_t, z_t, y_t, s) in period t . Then the distribution of people across cities, $L_t(z)$ is given by

$$\int_{\underline{z}}^z L_t(z) H(dz) = \sum_{i=1}^N \int_{\underline{a}}^{\infty} \int_{\underline{z}}^{\bar{z}} \int_{\underline{s}}^{\bar{s}} \mathbf{1}[z^*(a, z, y_i, s) \leq z] F_t(da, dz, ds) \quad \text{for all } z \in [\underline{z}, \bar{z}]$$

and rents are given by $q(z) = Q(L(z))$. This economy converges to a steady state where the distribution F_t is constant over time.

An equilibrium of the model above can be computed numerically. We do so for a CRRA utility function,

¹¹It is feasible to relax this restriction and introduce idiosyncratic skill shocks, although at some computational cost.

for a uniform distribution of cities, and for a particular house rent schedule.¹² We choose reasonable parameters values that allow us to illustrate the main forces at work. The exact values, specifications, and solution method are described in Appendix B.

Figure 4 presents the results of a simulation of this model. We focus on the reaction of a particular individual to a transitory income shock. The figure presents five panels, each of them displaying a different variable. For comparison purposes we present the behavior of an individual that can move (solid dark lines), and therefore use the location asset, and the behavior of an individual that cannot move from her preferred location when unconstrained, $\mathcal{Z}^U(s)$ (dashed light lines). The difference between these two cases represents the way in which the location asset helps the individual deal with the transitory income shock. We plot the effects for a particular individual with a fixed skill level.

The first panel in Figure 4 simply plots the income shock over time. The agent can be in two income states: high, $y_H = 0.5$, and low, $y_L = 0.1$.¹³ In period zero, the agent transitions from the high to the low income level. It stays there until the eighth period when he transitions back to the high income. This income process is identical for both scenarios, with and without mobility.

The second panel plots the level of financial assets. We start the individual at assets that are 120% of the transitory income level in the high state (the level of the maximum accumulated financial assets for the agent that cannot move). The individual also receives an income proportional to her skill and the city where she lived, $z_t s$. This additional income represents most of the individual's income. The transitory path represents between 15 and 20% of the agent's total income. As a result of the shock, the agent consumes part of her financial assets and therefore the asset balance declines until it hits zero, which is the level of the financial constraint. That is, individuals cannot borrow at all in financial markets. This decline in financial assets happens a bit faster when individuals can use the location asset, since in that case they know that when they hit the financial constraint they will be able to smooth consumption by moving. In period 2, the agent that cannot move hits the borrowing constraint and stays there for several periods. The agent that can use the location asset hits the borrowing constraint one period later. When the income shock reverses in period 8, without the location asset, the agent immediately starts saving and building a financial asset stock. In contrast, because at that point the location asset pays a higher return than the financial asset, the individual that can use the location asset, uses it to save. Such an individual stays stuck at the constraint for an extra two periods while it moves to better locations. Eventually, she reaches her desired location, the return she perceives on the location asset goes down, and she starts saving with the financial asset. Note that the presence of the location makes the individual stay longer at the financial constraint!

The third panel plots the location of the agent over time. The ideal unconstrained location of the agent is at city $\mathcal{Z}^U(s) = 0.77$. The agent that cannot move simply stays there throughout. The one that can

¹²In principle, specifying a given house rent schedule is without loss of generality, because we can find a skill distribution that would lead to this particular house rent schedule as an equilibrium outcome. Of course, endogenizing the house rent schedule is essential to perform aggregate counterfactual simulations. In the exercises below, we only consider counterfactuals that change the state of a particular individual and therefore do not affect the aggregate equilibrium allocation and prices.

¹³We could think of the low state as unemployment, and the high state as employment. Our calibration of the transition matrix Λ then implies a steady-state unemployment rate of about 6%.

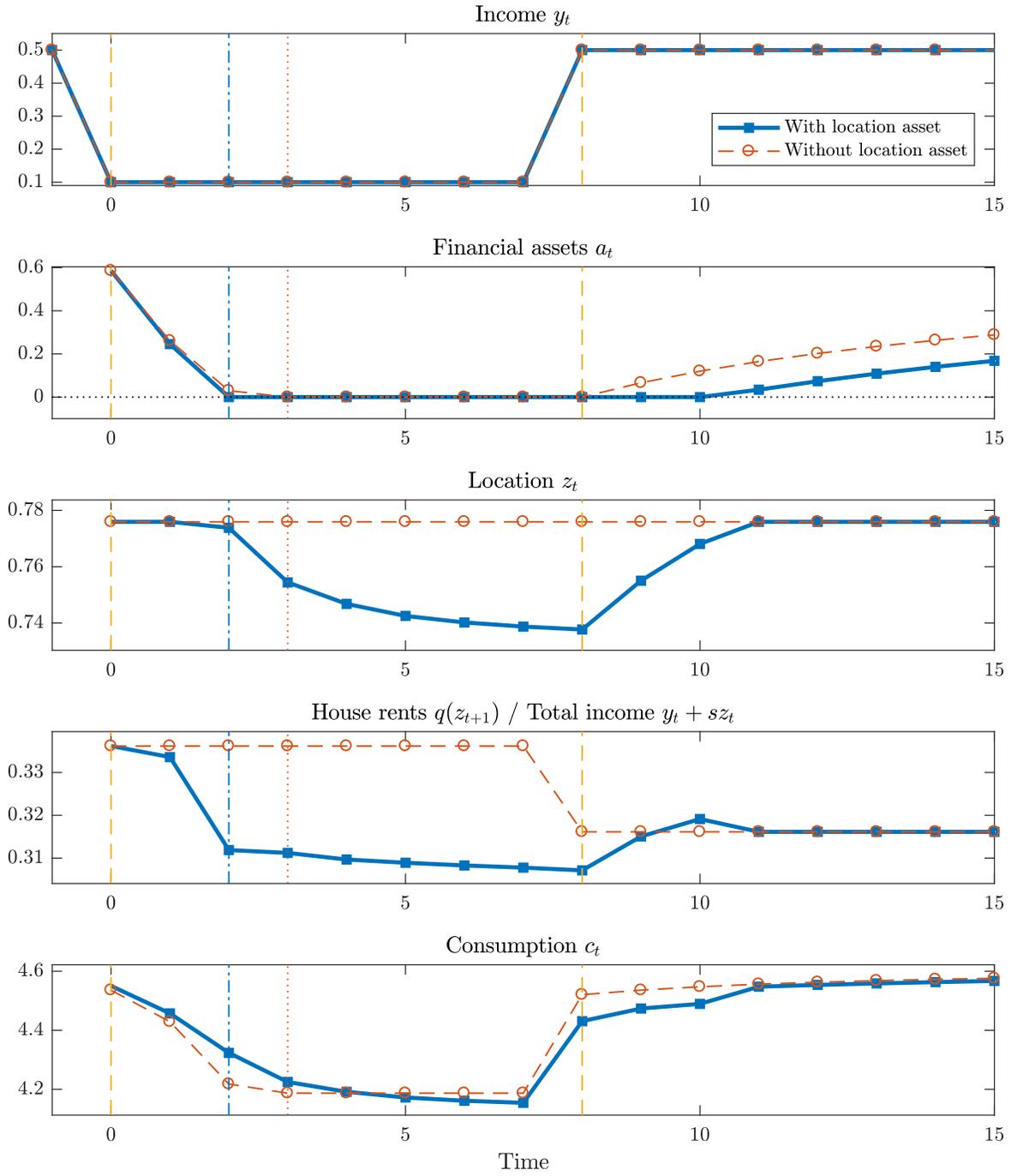


Figure 4: Dynamic Reaction from a Temporary Income Shock

stays there until financial assets hit the financial constraint. Once she runs down financial assets to zero, she starts borrowing using the location asset. That is, she starts downgrading her location progressively.

In this case the total downgrade is about 8%, similar in magnitude to the effects we calculate for France in the next section. This downgrading continues until the agent either reaches the minimum location she is willing to live in, or the shock reverses. In the plot it continues throughout the 7 periods where the agent obtains a low income. After the shock reverses to the high income state, the agent starts upgrading her location progressively. The last period where she is financially constrained, she reaches her unconstrained preferred city and starts saving with the financial asset only. Throughout the transition, the change in the city component of income due to the use of the location asset reaches 0.32, which lies in between the idiosyncratic income states.

The fourth panel in Figure 4 shows the share of housing expenditure in total income. The average share is about one third, similar to the data (Davis and Ortalo-Magne, 2011). At impact, the share of housing expenditure jumps up due to the fall in income. It starts falling from that peak since agents pay rents one period in advance. It keeps falling as the agents borrow with the location asset and downgrade their location. It falls even more as the income shock reverts to the high state and then starts increasing when the agent starts saving with the location asset. It overshoots the desired unconstrained level with high income due to the need to pay rents in advance, but eventually stabilizes at the same level as the immobile agent.

The bottom panel in the figure shows the agent consumption path with and without mobility. As we have underscored, the use of the location asset allows the agent to smooth consumption better since it can borrow even when she is at the financial constraint. The result is a consumption path that declines more slowly than without the location asset. Because borrowing with the location asset involves sacrificing future income, the total fall in consumption is also eventually somewhat larger. Once the shock reverses, the path out of the consumption slump is also smoother for agents that can use the location asset. Overall, these dynamic patterns of behavior vary substantially with and without the location asset .

The ability to use the location asset results in expected welfare gains for the agent.¹⁴ The presence of some gains is obvious given that the location asset provides a way of relaxing the friction imposed by the financial constraint and the agent can always decide not to move. In Figure 5 we present the percentage gain in consumption equivalent welfare from using the location asset. The values are calculated starting from the ideal city for unconstrained individuals, $Z^U(s)$, and we keep the skill of the individual fixed, as in Figure 4. Figure 5 then plots the relative welfare from using the location asset as a function of the starting asset level. It presents the gains for agents with a current high or low income realization. Clearly, because we are not estimating the parameters of the model for a particular circumstance, the level of the gains provides only a rough indication of what is at stake from using the location asset. In contrast, the qualitative patterns are more interesting. The benefits from using the location asset are as high as 3.7% close to the constraint. The welfare gains from mobility tend to decline with the level of assets and converge to zero for agents that are

¹⁴The gains from using the location asset for one particular path of realizations can be either negative or positive. For example, in Figure 4, the negative shock lasts for several periods. This increases the set of periods where agents that use the location asset obtain less consumption. However, this particular path is relatively unlikely. Other paths with shorter duration of the negative transitory shock yield larger benefits from the use of the location asset, and are more likely. In expectation, there are gains since the agent has a larger more flexible choice set.

immensely wealthy. The monotonicity with asset holdings holds throughout, except when asset holdings are close to the financial constraint of zero. Agents right at the constraint benefit less from the location asset than agents with small levels of wealth. The reason is that part of the benefits of the location asset comes from allowing individuals to smooth their consumption path as they dissave financial assets. Being right at the constraint eliminates these additional gains. The figure also shows that agents in the high income state benefit more than agents in the low income state. The reason is that the location asset allows individuals to consume more and save less in that state, since she aims to accumulate less of a precautionary asset stock.

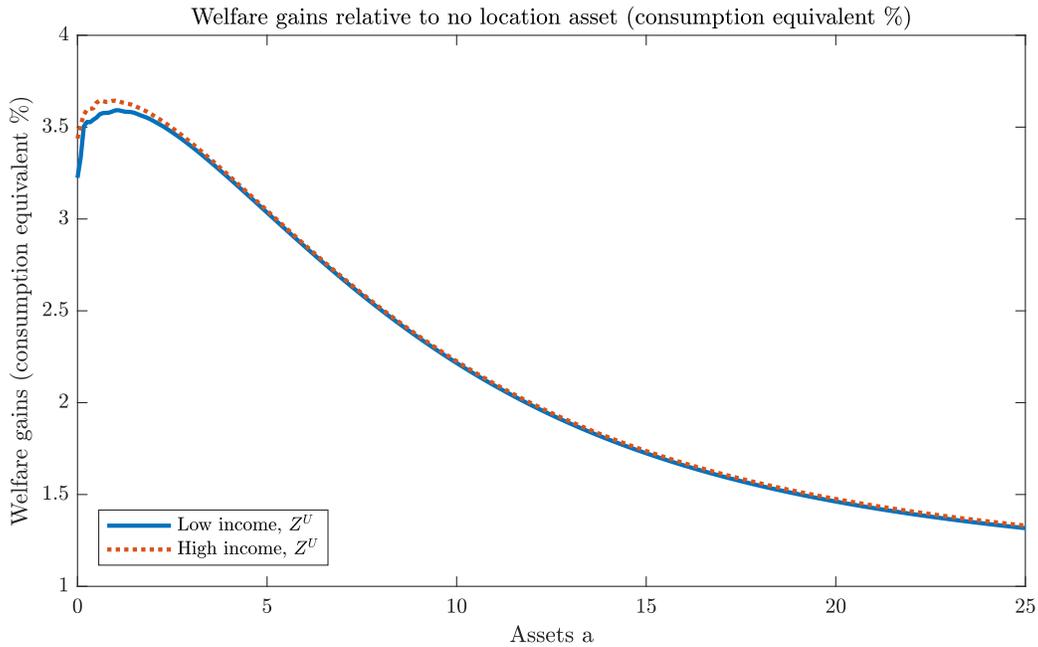


Figure 5: Welfare Gains from the Use of the Location Asset

Of course, because we have assumed that there is no cost of mobility at all, in our model agents optimize their location every period. Small moving costs would make adjustments to the agents' location, and therefore borrowing and saving with the location asset, more infrequent (although still beneficial). In addition, because the borrowing constraint generates a concave value function in wealth, small moving costs would reduce the frequency of moves more for low-wealth individuals relative to high-wealth individuals. Together with shocks to skill this would help explain jointly the sorting patterns across space and mobility patterns across income groups. We now explore some of the implications of our view of location choices using French individual location histories.

4 Location Choices in France

We have discussed in detail several implications of our view of location decisions as investing in a location asset. The main one of them is that constrained individuals will downgrade their location as a result of a negative front-loaded income shock. In contrast, unconstrained individuals will not react to these shocks. In this section we want to contrast this prediction with individual level data. We do so using data for France for the period 2002-2007. We use employer tax return data for all workers in the French economy. This is a short panel that identifies workers over two-year periods. Our data includes a worker identifier, the worker wages, the start and end dates of all her employment spells, residence municipality, as well as a number of worker characteristics like age, gender, occupation and birthplace. There are 36569 municipalities in France, with an average area of 15 squared kilometers and 435 inhabitants. Our dataset also includes a house price index for 101 regions (département) in France.

Our data is very detailed, however, contrasting it with our theoretical predictions involves several choices. First, since the data does not have information on assets or the extent to which workers are ‘constrained’ to borrow in financial markets, we need to take a stand on what are the worker’s characteristics that make them more likely to be constrained. Furthermore, we do not have the worker’s skill or her level of education.

Finally, we do not have a location characteristics that tells us which locations are more complementary with skill, or more attractive. To address these challenges we use our theoretical model. First, the positive assortative matching implication links a worker’s skill with her earnings which we observe. Furthermore, as implied by the model in Section 2, residents of cities with higher z will have higher average incomes. Hence, we can determine the z -rank of cities using the rank of their average income (see Figure 1). Finally, the model tells us that the highest income individuals in any city are the ones that are not constrained. Hence we can look at the percentile of an individual in the income distribution of her residence municipality to obtain an index of the likelihood that the individual is constrained (see Figure 2). Armed with these choices we are ready to explore the mobility choices of individuals and contrast them with the implications of our ‘location as an asset’ view.

We study the changes in residential locations as a result of an unemployment spell. We see the location of an individual in a particular job. As a result of job termination the individual disappears from the dataset and we see the individual appear again when she find another job. We select individuals that have exactly one unemployment spell of at least 40 days in a 2-year period, that had employment before and after for at least 90 days, and that switch employer after the unemployment spell. This selection guarantees that the shock is significant in magnitude and avoids people that have unstable temporary employment. After an individual find a new job, we can look at the average income rank of the new residential location and compare it with the original rank.¹⁵

Figure 6 provides some basic statistics for our data. The top panel plots the number of individuals that go through unemployment spells of different lengths from one to four quarters. Our data includes more that

¹⁵Section C.1 contains a thorough description of the data and the construction of the sample.

two and a half million individuals that go through one and two quarter long unemployment spells, about a million and one quarter that go through a three quarter spell, and a much smaller number that go through unemployment spells longer than that. For comparison purposes, about two million individuals change jobs without an unemployment spell in between. So there is plenty of data to study the type of transitions we are interested in. The bottom panel presents the fraction of agents that change location as a function of the length of the unemployment spell. It distinguishes between agents in the bottom quintiles of the income distribution, top quintile, and all agents. The fraction of movers that are unemployed for one quarter or change jobs without going through unemployment is above 20%. The fraction increases with the length of the unemployment spell, and between the top and bottom income quintile, although it is not monotone in income for all unemployment spell lengths. Thus, the number of agents that see a change in employment and move is substantial, particularly if they go through a long unemployment spell.¹⁶

The main implication of our model is that individuals with a low income rank in their original location (and therefore presumably financially constrained according to our theory) should downgrade their location relative to individuals in the same location who are at the top of the location’s income distribution. Therefore we estimate the following regression,

$$P(z_{1it}) - P(z_{0it}) = \alpha_{z_{0t}} + \alpha_I + \beta_w P(w_{it}; z_{0it}) + \varepsilon_{it}, \tag{9}$$

where $P(z_{1it})$ is the percentile of the origin municipality in the separation year, $P(z_{0it})$ is the percentile of the destination municipality in the job finding year, $\alpha_{z_{0t}}$ denotes municipality-time fixed effects, α_I denotes a set of worker characteristics fixed effects (e.g. age, gender, birthplace, occupation), $P(w_{0it}; z_{0it})$ is individual i ’s income percentile in municipality z_{0it} , and ε_{it} is a mean zero error term that we assume has the standard mean independence properties. We are particularly interested in the value of β_w . The theory predicts that agents that are lower in the income rank of their origin municipality should downgrade relative to others as a result of the unemployment spell. So our ‘location as an asset’ view implies that $\beta_w > 0$.

Table 1 presents the results for β_w when we select the sample to agents that move as a result of (or concurrently to) the unemployment shock.¹⁷ As implied by our view, the estimated coefficient on the origin wage percentile is positive and significant. The magnitude varies between 0.089 and 0.185 depending on the set of fixed effects we use in the regression. All standard errors are clustered at the département level. The coefficient increases as we add a more and more complete set of individual characteristic fixed effects. The interpretation is simple, using our preferred estimate in column four, a job termination implies that agents in the bottom percentile of their location’s income distribution downgrade to a location 16.5 percentile points worse than the highest-income agents in their original location. This is a large effect that indicates very different mobility patterns across individuals. The results in column four should be interpreted as the effect for workers with the same original municipality, time period, age, gender, birthplace, and in the same

¹⁶Long unemployment spells are also less likely to reflect voluntary vacation periods between jobs.

¹⁷The table is computed using city ranks that are allowed to change yearly, although the results using fixed city ranks calculated at the beginning of the sample are virtually identical. We show the latter in Appendix C.2

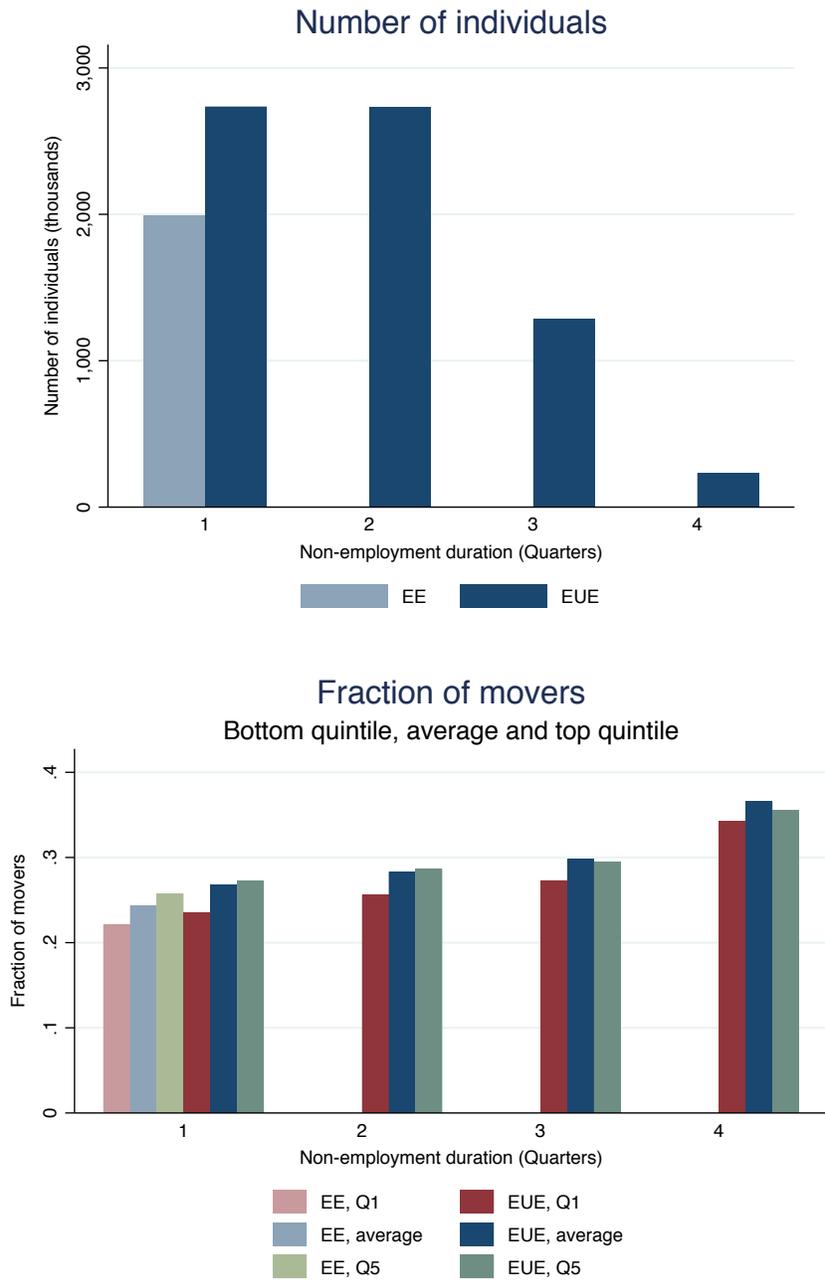


Figure 6: Descriptive Statistics

pre-separation 2-digit occupation and industry. In column five we use a more detailed 4-digit classification and industry and the results grow marginally to 18.5 percentile points.

Comparing columns one to three with columns four and five, it is clear that industry and occupation

Table 1: Unemployment Spells and Location Decisions: Movers

Yearly City Rank. Movers only.					
Origin Wage Percentile (OWP)	0.089*** (0.008)	0.101*** (0.006)	0.103*** (0.006)	0.165*** (0.005)	0.185*** (0.006)
Constant	-0.047*** (0.009)				
<i>Fixed Effects</i>					
Municipality-Year		✓	✓	✓	✓
Age			✓	✓	✓
Birthplace			✓	✓	✓
Gender			✓	✓	✓
2-Digit Origin Occupation				✓	
2-Digit Origin Industry				✓	
4-Digit Origin Occupation					✓
4-Digit Origin Industry					✓
Obs.	1965989	1925294	1920247	1379825	1378926
R^2	0.005	0.080	0.082	0.097	0.105
W.- R^2		0.006	0.007	0.013	0.015

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
26,868 Origin Municipalities; 2003-2007.

fixed effects increase the magnitude of the coefficients significantly. This is probably related to the fact that occupations and industries are clustered in space. As a result, in the data individuals in the same occupation and industry with very different incomes tend to move to the same location due to the spatial concentration of labor demand in specific industries and occupations. Our focus is on the effect of financial constraints on consumption smoothing through financial and location assets, so adding these fixed effects is preferable. Similarly, life cycle effects and historical ties to certain locations might affect the choices of individuals beyond the forces in our model. Hence, adding these fixed effects is probably important as well, although in practice this addition does not change the estimates much. Independently of the specification chosen, our main hypothesis is clearly not falsified by these results.

Figure 7 presents the change in location percentile for agents at the bottom quintile, relative to all other quintiles. It shows our estimates when we estimate effects separately for agents that exhibit different unemployment spell lengths. The top panel presents the effect for all individuals, including movers and non-movers, while the bottom panel presents the effects of movers. The graph shows that the main effects of unemployment on location are similar for unemployment spells between 1 and 3 quarters, while they

are a bit larger for individuals that remain a whole year unemployed. The effects are monotone in the agent's income quintile in the original location, as can be determined by the shifting down of the curves as we compare with higher quintiles. This is exactly what we would expect if initial income percentile makes borrowing constraints more binding.

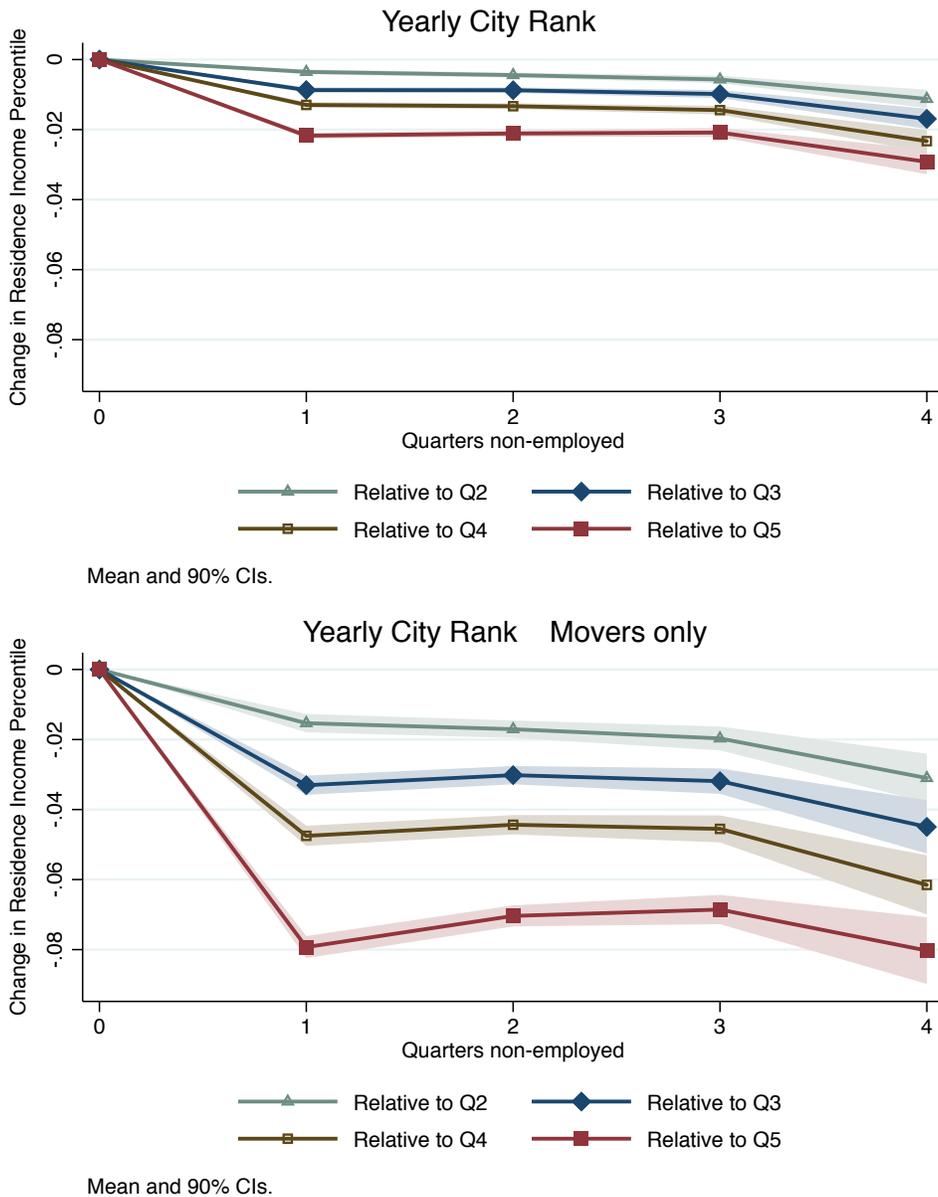


Figure 7: Change in Residence Percentile and Unemployment Spell Length

One potential concern with the results above is that individuals at the bottom of the income distribution in their original location tend to go to lower ranked locations relative to other individuals for reasons

unrelated to the unemployment shock. For example, they could be progressively optimizing their location by correcting past location mistakes. That is, the results above could be simply capturing some form of mean reversion in the data that controlling the variety of fixed effects in Table 1 does not eliminate. To address this potential concern, ideally we would need a comparison group that moves but does not have an incentive to use the location asset to borrow. Finding such a comparison group is hard because the desire to borrow depends on the whole future expected income path. This expectation is likely affected by any job transition or move. We make an attempt to find a suitable, although not perfect, comparison by pulling all employment to employment transitions that generate moves and positive wage growth, together with all transitions through non-employment that generate at least a 25% decline in wages. We then estimate the differential effect of origin wage percentile for workers that go through an unemployment spell. The conditioning on wage growth for job-to-job transitions eliminates some of the transitions that would generate location downgrading due to the desire to borrow. The conditioning on wage decline for the agents that go through unemployment spells makes the desire to borrow for this group larger.

The results are presented in Table 2. The first row indicates the overall effect of origin wage percentile, while the second row indicates the differential effect. We expect the differential effect to be positive and significant. This is the case for all combinations of fixed effects presented in the table. In our preferred specification in column four, the differential effect is a downgrade of 4 percentage points for individuals at the bottom of the origin wage percentile (relative to those at the top) that go through an unemployment spell relative to those that go through a job-to-job transition. The finding in the first row that the effect of origin wage percentile is large and significant indicates that agents in this sub-sample downgrade more if they have relatively low wages in their location. This might be the result of the cost involved in moves or job switches, or the benefit of the job switch in terms of higher wage growth in the future (which makes individuals want to borrow with the location asset even in job-to-job transitions that increase wages).

Table 3 presents the set of results in Table 1 but when we include the wage growth percentile (WGP) and the correlation between wage growth percentile and origin wage percentile (OWP). Wage growth percentile is the percentile of the observed real wage change in the distribution of real wage changes. We compute real wages dividing the nominal wage by the average house prices in the relevant département. Namely, we estimate

$$\begin{aligned}
 P(z_{1it}) - P(z_{0it}) &= \alpha_{z_{0t}} + \alpha_I + \beta_w P(w_{0it}; z_{0it}) + \\
 &\quad \gamma P(w_{1it}/w_{0it}) + \delta P(w_{0it}; z_{0it}) P(w_{1it}/w_{0it}) + \varepsilon_{it},
 \end{aligned}
 \tag{10}$$

where $P(w_{1it}/w_{0it})$ denotes the WGP.

Although we would hypothesize that most wage changes will be related to the agent's location choice, this specification recognizes that workers might obtain wage offers that are particularly high or low for idiosyncratic reasons. In particular, workers may have advance information regarding the future idiosyncratic component of wages. In addition, if wages are mean-reverting, then workers who anticipate and experience

Table 2: Location Decisions of Unemployed Relative to Job Switchers

Movers only. EUE transitions relative to EE transitions.				
Origin Wage Percentile (OWP)	0.110*** (0.008)	0.109*** (0.008)	0.122*** (0.006)	0.189*** (0.006)
OWP*1[EUE]	0.013** (0.004)	0.015** (0.005)	0.028*** (0.004)	0.042*** (0.006)
Constant	-0.052*** (0.008)			
Constant*1[EUE]	-0.054*** (0.003)			
<i>Fixed Effects</i>				
Municipality-Year		✓	✓	✓
Age			✓	✓
Birthplace			✓	✓
Gender			✓	✓
2-Digit Origin Occupation				✓
2-Digit Origin Industry				✓
Obs.	1212774	1142728	1128360	830710
R^2	0.011	0.111	0.113	0.131
W.- R^2		0.008	0.010	0.017

Standard errors in parentheses, clustered at the departement level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. 26,868 Origin Municipalities, 2003-2007.

EE transitions condition on positive wage growth, EUE transitions on a 25% wage decline.

All fixed effects vary by transition type (EE and EUE).

low wage growth today also anticipate and experience high wage growth tomorrow. Thus, these workers want to save using the ‘location asset’. Hence, a particularly good realization of the idiosyncratic component of wages should be accompanied by location upgrading, while a particularly bad one should come with location downgrading.

Our theory also adds an endogenous feedback to this first-round effect. Indeed, realized wages should respond to location choice on top of the exogenous idiosyncratic component of income. If individuals start downgrading because of advance information, they also lower their expected income since z_{t+1} falls. This in turn magnifies the positive link between location choice and wage growth. Thus, we expect $\gamma > 0$.¹⁸

In fact, the estimates in Table 3 yield a positive and significant estimate of around 0.1, with somewhat larger estimates when we use industry and occupation fixed effects. These results imply that workers that

¹⁸Naturally, if location amenities are normal goods, higher income would also drive individuals to move to more expensive locations with better amenities, a mechanism that we do not explicitly model.

Table 3: Unemployment Spells and Location Decisions: Movers and Observed Wage Growth

Yearly City Rank. Movers only. Observed wages.					
Origin Wage Percentile (OWP)	0.153*** (0.017)	0.149*** (0.015)	0.177*** (0.013)	0.256*** (0.010)	0.279*** (0.010)
Wage Growth Percentile (WGP)	0.098*** (0.013)	0.098*** (0.012)	0.105*** (0.012)	0.125*** (0.008)	0.127*** (0.008)
OWP*GWP	-0.031* (0.012)	-0.010 (0.011)	-0.020* (0.010)	-0.034** (0.012)	-0.029* (0.012)
Constant	-0.121*** (0.014)				
<i>Fixed Effects</i>					
Municipality-Year		✓	✓	✓	✓
Age			✓	✓	✓
Birthplace			✓	✓	✓
Gender			✓	✓	✓
2-Digit Origin Occupation				✓	
2-Digit Origin Industry				✓	
4-Digit Origin Occupation					✓
4-Digit Origin Industry					✓
Obs.	1966030	1933747	1920410	1379947	1379020
R^2	0.010	0.084	0.088	0.105	0.113
W.- R^2		0.012	0.013	0.021	0.023

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
26,868 Origin Municipalities; 2003-2007.

Note: Number of observations differs between regressions without and with wage controls because of missing characteristics.

experience the largest wage growth upgrade city by about 10 percentile points relative to workers that obtain the smallest increase in wages. We emphasize again that this is just an equilibrium relationship. In no way are we proposing a causal interpretation of these results. Choosing better locations might result in better jobs, but better jobs might also incentivate a move to a better location. Both channels are probably active and significant. In any case the addition of WGP does not reduce the effect of the agent's OWP. If anything, by absorbing the direct relationship between wage growth and location it makes the effect of financial constraints on location even larger. In the last two columns, with a complete set of fixed effects, the difference between the top and bottom percentile of the income distribution in the original location is as large as 25 or 28 percentile points.

The regression in (10) also includes an interaction term. The sign of this interaction indicates whether the effect of financial constraints is larger or smaller for agents that experience large or small wage changes. The location as an asset view tells us that agents that are constrained, and use the location asset more, go to worse locations where rents are cheaper and future job and education prospects are worse. Hence, in equilibrium, more constrained agents that obtain lower wage growth should be the ones that downgrade location.¹⁹

Therefore, the implication of the ‘location as an asset’ view is that $\delta < 0$. This is what we find in Table 3, although the interaction effect is not always significant. Note that in our argument wages are partly the outcome of the location choice not the sole cause of it. The direct effect that causes changes in location is captured by γ and does not interact with the level of financial constraints.

In Table 3 we use directly observed wages, to calculate the real wage growth numbers that underlie the WGP variable. As it is common in the literature, one might prefer to use wage residuals after controlling for a number of individual characteristics. In Table 4 we use residual wages after discarding age, gender, and two digit occupation and industry components of wages (results when we control for four digit fixed affects are similar). That is, we use a measure of wage growth that is more obviously idiosyncratic or the result of the agent’s location choice. The results in Table 4 are very similar, but all larger in absolute value and more significant than those in Table 3. The interaction terms is now negative and large and the direct effect of OWP grows to almost 30 percentile points. Overall, this constitutes evidence that the ‘location as an asset’ view can rationalize big differences in behavior between constrained and unconstrained individuals. Furthermore, if we drop municipality time fixed effects so that we can estimate the level of the location choices, the results in column one indicate that an individual that is at the bottom of the origin income distribution and finds a job that results in the lowest possible wage growth will go to a location that is 12 percentage points worse than the original location where she started.

We finish this section with a similar exercise where instead of using only workers that move concurrently to the unemployment spell we use all workers, including the ones that do not move. We still use residual wages calculated as in Table 4. Table 5 presents these results. Naturally, since many individuals do not move, the results are much smaller in magnitude. On average, as we saw in Figure 6 only about a quarter of workers move and so the new results are about a quarter as large as the previous ones. Nevertheless, they are all significant. In levels, with an average wage growth shock, unconstrained individuals at the top of the distribution upgrade about 2.3 percentile points while constrained individuals at the bottom of the distribution downgrade by about 3.3 percentage points. When we add the battery of fixed effects the difference grows to almost 10 percentage points for individuals with average wage growth. This is an extremely large differential effect of unemployment on individual location choices. One that is well rationalized by our ‘location as an asset’ view.

¹⁹Again, under private advance information and mean-reversion, we would expect high-wealth individuals to react less to wage growth because they have enough assets to stay at their preferred location, irrespectively of their expectations of future wages. In contrast, low-wealth individuals’ location choices are more sensitive to signals regarding future wages. Any signal predicting low current wages (and hence high future wage growth) should lead to a downward move.

Table 4: Unemployment Spells and Location Decisions: Movers and Residual Wage Growth

Yearly City Rank. Movers Only.					
Wages net of age, gender, occupation and industry fixed effects.					
Origin Wage Percentile (OWP)	0.194*** (0.013)	0.194*** (0.014)	0.231*** (0.011)	0.297*** (0.014)	0.326*** (0.015)
Wage Growth Percentile (WGP)	0.097*** (0.010)	0.101*** (0.011)	0.109*** (0.010)	0.137*** (0.012)	0.143*** (0.012)
OWP*WGP	-0.104*** (0.009)	-0.096*** (0.008)	-0.099*** (0.008)	-0.077*** (0.009)	-0.073*** (0.009)
Constant	-0.122*** (0.011)				
<i>Fixed Effects</i>					
Municipality-Year		✓	✓	✓	✓
Age			✓	✓	✓
Birthplace			✓	✓	✓
Gender			✓	✓	✓
2-Digit Origin Occupation				✓	
2-Digit Origin Industry				✓	
4-Digit Origin Occupation					✓
4-Digit Origin Industry					✓
Obs.	888834	852845	850755	850696	849804
R^2	0.011	0.099	0.104	0.112	0.123
W.- R^2		0.012	0.014	0.021	0.024

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
26,868 Origin Municipalities; 2003-2007.

Note: Number of observations differs between regressions without and with wage controls because of missing characteristics.

5 Conclusions

This paper provides an alternative view of individual location decisions. We have argued that we can understand location decisions as an investment that allows individuals to transfer resources across periods even when they are constrained in financial markets. Individuals that are constrained to borrow in the financial markets use the location asset to borrow and live in locations that offer relatively bad work and educational opportunities but are cheap in terms of housing costs and other local expenses. Hence, our view of location choices underscores the importance of the incentives to smooth consumption and the extent to which individuals face financial constraints as essential to understand where they live.

Table 5: Unemployment Spells and Location Decisions: All Individuals and Residual Wages

Yearly City Rank. All individuals. Wages net of age, gender, occupation and industry fixed effects.

Origin Wage Percentile (OWP)	0.026*** (0.003)	0.027*** (0.002)	0.048*** (0.002)	0.054*** (0.002)	0.056*** (0.004)	0.053*** (0.004)	0.084*** (0.003)	0.093*** (0.003)
Wage Growth Percentile (WGP)					0.028*** (0.002)	0.027*** (0.003)	0.037*** (0.002)	0.039*** (0.002)
WGP*WGP					-0.029*** (0.003)	-0.023*** (0.003)	-0.013** (0.004)	-0.011** (0.004)
Constant	-0.013*** (0.002)				-0.033*** (0.002)			
<i>Fixed Effects</i>								
Municipality-Year		✓	✓	✓		✓	✓	✓
Age			✓	✓			✓	✓
Birthplace			✓	✓			✓	✓
Gender			✓	✓			✓	✓
2-Digit Origin Occupation			✓				✓	
2-Digit Origin Industry			✓				✓	
4-Digit Origin Occupation				✓				✓
4-Digit Origin Industry				✓				✓
Obs.	6965851	6916411	4905080	4904221	3033008	3010760	3003581	3002732
R^2	0.002	0.149	0.151	0.154	0.003	0.133	0.137	0.140
W.- R^2		0.002	0.004	0.04		0.003	0.006	0.007

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

26,868 Origin Municipalities; 2003-2007.

Note: Number of observations differs between regressions without and with wage controls because of missing characteristics.

We show that the implications of our model can rationalize the moving choices observed in France when individuals go through unemployment spells. More generally, our view can help explain why some individual locate in areas that seem so undesirable otherwise. The fact that many individuals decide choose to live in such locations rather than in areas that offer more opportunities, might seem puzzling from a static perspective, but is a perfectly reasonable choice through the lens of our dynamic theory. In most cases the previous literature has relied on unobserved migration costs to explain these choices. In contrast, our view rationalizes this behavior even when migration is perfectly free. The change in perspective is relevant for policy. As we have argued, using place-based policies to improve some of the worse locations can harm some of the less skilled agents in the economy.

Of course, the location as an asset view is more general than the particular model we put forward in

this paper and can be contrasted more fully with the data. For example, modelling location choices in an overlapping generations model with locations could help us understand the implications of our view for life-cycle patterns and investment in the skills of descendants. Modelling location choices as changing the properties of an agents income process (by, for example, affecting the likelihood of becoming unemployed) would allow us to study the value of the location asset to manage risk. Finally, embedding this type of consumption-savings decision with borrowing constraints in a fully-fledged quantitative spatial model with skill complementarities, factor price determination, mobility and trade could help decompose the role of the location asset in determining net mobility patterns relative to other forces. It could also help us understand how the use of location as an asset affects the evaluation of global phenomena that affect factor rewards in particular locations, occupations and industries.

References

- [1] T. Allen and C. Arkolakis. Trade and the Topography of the Spatial Economy. Working Paper 19181, National Bureau of Economic Research, June 2013.
- [2] J. Altonji and R. Mansfield. The Role of Family, School, and Community Characteristics in Inequality in Education and Labor-Market Outcomes. Technical report, 2011.
- [3] G.-M. Angeletos. Uninsured Idiosyncratic Investment Risk and Aggregate Saving. *Review of Economic Dynamics*, 10(1):1 – 30, 2007.
- [4] L. Caliendo, M. Dvorkin, and F. Parro. Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock. Working Papers 2015-9, Federal Reserve Bank of St. Louis, May 2015.
- [5] L. Caliendo, L. D. Opromolla, F. Parro, and A. Sforza. Goods and Factor Market Integration: A Quantitative Assessment of the EU Enlargement. Working Paper 23695, National Bureau of Economic Research, August 2017.
- [6] C. D. Carroll. The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems. *Economics Letters*, 91(3):312 – 320, 2006.
- [7] P. Cavalcanti-Ferreira, A. Monge-Naranjo, and L. T. de Mello Pereira. Of Cities and Slums. Technical report, 2018.
- [8] R. Chetty and N. Hendren. The Impacts of Neighborhoods on Intergenerational Mobility I: Childhood Exposure Effects. Working Paper 23001, National Bureau of Economic Research, December 2016.

- [9] R. Chetty, N. Hendren, and L. F. Katz. The Effects of Exposure to Better Neighborhoods on Children: New Evidence from the Moving to Opportunity Experiment. *American Economic Review*, 106(4):855–902, April 2016.
- [10] D. M. Cutler and E. L. Glaeser. Are Ghettos Good or Bad? *The Quarterly Journal of Economics*, 112(3):827–872, 1997.
- [11] M. A. Davis, J. Gregory, D. A. Hartley, and K. Tan. Neighborhood Choices, Neighborhood Effects and Housing Vouchers. FRB of Chicago Working Paper No. WP-2017-2, 2015.
- [12] M. A. Davis and F. Ortalo-Magne. Household Expenditures, Wages, Rents. *Review of Economic Dynamics*, 14(2):248 – 261, 2011.
- [13] N. A. Denton and D. S. Massey. *American Apartheid: Segregation and the Making of the Underclass*. Cambridge, Mass: Harvard University Press.
- [14] K. Desmet, D. Nagy, and E. Rossi-Hansberg. The Geography of Development. *Journal of Political Economy*, 2018.
- [15] K. Desmet and E. Rossi-Hansberg. Urban Accounting and Welfare. *American Economic Review*, 103(6):2296–2327, October 2013.
- [16] K. Desmet and E. Rossi-Hansberg. On the Spatial Economic Impact of Global Warming. *Journal of Urban Economics*, 88:16 – 37, 2015.
- [17] R. Diamond. The Determinants and Welfare Implications of US Workers’ Diverging Location Choices by Skill: 1980-2000. *American Economic Review*, 106(3):479–524, March 2016.
- [18] R. Diamond, T. McQuade, and F. Qian. The Effects of Rent Control Expansion on Tenants, Landlords, and Inequality: Evidence from San Francisco. Working Paper 24181, National Bureau of Economic Research, January 2018.
- [19] G. J. Duncan, J. Ludwig, L. A. Gennetian, L. F. Katz, R. C. Kessler, J. R. Kling, and L. Sanbonmatsu. Long-Term Neighborhood Effects on Low-Income Families: Evidence from Moving to Opportunity. *American Economic Review*, 103(3):226–31, May 2013.
- [20] R. Fernandez and R. Rogerson. Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform. *The American Economic Review*, 88(4):813–833, 1998.
- [21] A. Fogli and V. Guerrieri. The End of the American Dream? Inequality and Segregation in US cities. Technical report, 2017.
- [22] A. Galichon. *Optimal Transport Methods in Economics*. Princeton University Press, 2016.

- [23] E. Giannone. Skill-Biased Technical Change and Regional Convergence. 2017 Meeting Papers 190, Society for Economic Dynamics, 2017.
- [24] C.-T. Hsieh and E. Moretti. Housing Constraints and Spatial Misallocation. Working Paper 21154, National Bureau of Economic Research, May 2015.
- [25] M. Huggett. The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies. *Journal of Economic Dynamics and Control*, 17(5):953 – 969, 1993.
- [26] G. Kaplan and S. Schulhofer-Wohl. Understanding the Long-Run Decline in Interstate Migration. *International Economic Review*, 58(1):57–94, 2017.
- [27] J. Kennan and J. R. Walker. The Effect of Expected Income on Individual Migration Decisions. *Econometrica*, 79(1):211–251, 2011.
- [28] R. E. J. Lucas. Life Earnings and Rural-Urban Migration. *Journal of Political Economy*, 112(S1):S29–S59, 2004.
- [29] B. Moll. Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation? *American Economic Review*, 104(10):3186–3221, October 2014.
- [30] M. Morten. Temporary Migration and Endogenous Risk Sharing in Village India. Working Paper 22159, National Bureau of Economic Research, April 2016.
- [31] S. J. Redding. Goods Trade, Factor Mobility and Welfare. *Journal of International Economics*, 101:148 – 167, 2016.
- [32] L. A. Sjaastad. The Costs and Returns of Human Migration. *Journal of Political Economy*, 70(5):80–93, 1962.
- [33] W. J. Wilson. *The Truly Disadvantaged. The Inner City, the Underclass and Public Policy*. University of Chicago Press, 1987.

A Appendix: Proofs for the model in Section 2

A.1 Proof of Lemma 1

We split the proof in three parts:

1. Location decisions of constrained and unconstrained individuals
2. Equilibrium in cities in which at least one unconstrained individual lives
3. Equilibrium in cities with only constrained individuals

A.1.1 Location decisions

Recall that for unconstrained individuals,

$$R = \frac{s}{q'(z)}$$

Therefore, unconstrained individuals of skill s locate in cities $\mathcal{Z}^U(s)$ such that

$$R = \frac{s}{q'(\mathcal{Z}^U(s))}$$

In addition, some constrained individuals may choose cities in which only constrained individuals locate. For those individuals, we cannot use the expression above, and we directly use the migration Euler equation:

$$\frac{(y_1 + R\underline{a}) + z\mathcal{S}^C(y_0, y_1, z)}{\beta[y_0 - \underline{a} - q(z)]} = \frac{\mathcal{S}^C(y_0, y_1, z)}{q'(z)}$$

which implies

$$\mathcal{S}^C(y_0, y_1, z) = \frac{q'(z)(y_1 + R\underline{a})}{\beta[y_0 - \underline{a} - q(z)] - zq'(z)} \quad (11)$$

Notice that for constrained individuals $(y_0, y_1, \mathcal{S}^C(y_0, y_1, z))$ who locate in a city z where at least one unconstrained individual with skill $\mathcal{S}^U(z)$ lives, we can substitute out $q'(z) = \mathcal{S}^U(z)/R$, leading to

$$\mathcal{S}^C(y_0, y_1, z) = \frac{\mathcal{S}^U(z)(y_1 + R\underline{a})}{\beta R(y_0 - \underline{a} - q(z)) - z\mathcal{S}^U(z)} \quad (12)$$

In the sequel, it will be useful to have notation for this relationship in terms of all the endogenous objects. Therefore, we define

$$X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) = \frac{s(y_1 + R\underline{a})}{\beta R(y_0 - \underline{a} - q(\mathcal{Z}^U(s))) - \mathcal{Z}^U(s)s} \quad (13)$$

Equation (13) describes which constrained individuals $(y_0, y_1, X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))))$ choose to locate in city $\mathcal{Z}^U(s)$.

To obtain the lowest possible income in a given city, we can re-write equation (12) as

$$y_0 = \underline{a} + q(z) + \frac{1}{\beta R} \left[z \mathcal{S}^U(z) + \frac{(y_1 + R\underline{a}) \mathcal{S}^U(z)}{\mathcal{S}^C(y_0, y_1, z)} \right] \quad (14)$$

This delivers the lower bound on initial income for constrained individuals who locate in city z with at least an unconstrained individual:

$$y_0 \geq \underline{Y}_0(z) = \underline{a} + q(z) + \frac{1}{\beta R} \left[z \mathcal{S}^U(z) + (\underline{y}_1 + R\underline{a}) \times \frac{\mathcal{S}^U(z)}{\bar{s}} \right]$$

A similar bound involving $q'(z)$ holds for cities in which only unconstrained individuals live.

A.1.2 Equilibrium in cities with at least one unconstrained individual

We first consider equilibrium in cities with at least one constrained individual. Because at any skill, constrained individuals locate in worse cities than unconstrained individuals, cities with unconstrained individuals have higher z than those with only constrained individuals. Thus, there exists a cutoff \hat{z} such that a city has at least one unconstrained individual iff $z \geq \hat{z}$.

We start by assuming that the matching function $\mathcal{Z}^U(s)$ is increasing at all s . Total population that locates in cities $[\mathcal{Z}^U(s), \mathcal{Z}^U(s) + \mathcal{Z}_s^U(s) ds]$ is the sum of the unconstrained individuals of the same skill and constrained individuals of higher skill. Before expressing total population, we denote by

$$\tilde{A}(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) = y_0 - q(\mathcal{Z}^U(s)) - \frac{y_0 - q(\mathcal{Z}^U(s)) + \frac{y_1 + s \mathcal{Z}^U(s)}{R}}{1 + \beta}$$

desired savings as a function of individual characteristics and the matching function. Using the notation we defined, we can express total population as:

$$\begin{aligned} & G(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)), \mathcal{Z}_s^U(s)) \\ & \equiv \iint f(y_0, y_1, s) \mathbf{1} \left[\tilde{A}(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) > \underline{a} \right] dy_0 dy_1 \\ & + \iint \mathbf{1} \left[\tilde{A}(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \leq \underline{a} \right] \\ & \quad \times f(y_0, y_1, X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))) \times \frac{d[X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))]}{ds} dy_0 dy_1 \end{aligned}$$

where it is understood that $\frac{d[X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))]}{ds}$ is the total derivative of $s \mapsto X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))$

with respect to s . We can calculate this last term explicitly:

$$\begin{aligned} \frac{d[X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))]}{ds} &\equiv X_0(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \\ &+ \frac{X_1(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \times \mathcal{Z}_s^U(s)}{1 + \beta} \\ &+ \frac{\beta R X_1(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))}{s} \times \frac{q'(\mathcal{Z}^U(s)) \times \mathcal{Z}_s^U(s)}{1 + \beta} \end{aligned}$$

where an s subscript denotes a derivative w.r.t. s , and where we define

$$\begin{aligned} X_0(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) &= \frac{y_1 + R\underline{a}}{\beta R(y_0 - \underline{a} - q(\mathcal{Z}^U(s))) - \mathcal{Z}^U(s)s} + \frac{s\mathcal{Z}^U(s)(y_1 + R\underline{a})}{[\beta R(y_0 - \underline{a} - q(\mathcal{Z}^U(s))) - \mathcal{Z}^U(s)s]^2} \\ &= \frac{\beta R(y_0 - \underline{a} - q(\mathcal{Z}^U(s)))(y_1 + R\underline{a})}{[\beta R(y_0 - \underline{a} - q(\mathcal{Z}^U(s))) - \mathcal{Z}^U(s)s]^2} \end{aligned}$$

and

$$\frac{X_1(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))}{1 + \beta} = \frac{s^2 \mathcal{Z}^U(s)(y_1 + R\underline{a})}{[\beta R(y_0 - \underline{a} - q(\mathcal{Z}^U(s))) - \mathcal{Z}^U(s)s]^2}$$

We now make use once again of the migration Euler equation $q'(\mathcal{Z}^U(s)) = R/s$ to re-write

$$\begin{aligned} \frac{d[X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))]}{ds} &= X_0(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \\ &+ X_1(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \times \mathcal{Z}_s^U(s) \end{aligned}$$

Substituting these expressions into our expression for the supply of individuals in cities $[\mathcal{Z}^U(s), \mathcal{Z}^U(s) + \mathcal{Z}_s^U(s)ds]$, we obtain

$$G(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)), \mathcal{Z}_s^U(s)) = A(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) + B(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \times \mathcal{Z}_s^U(s)$$

where we defined

$$\begin{aligned} A(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) &= \iint f(y_0, y_1, s) \mathbf{1} \left[\tilde{A}(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) > \underline{a} \right] dy_0 dy_1 \\ &+ \iint \mathbf{1} \left[\tilde{A}(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \leq \underline{a} \right] \\ &\quad \times f(y_0, y_1, X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))) \\ &\quad \times X_0(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) dy_0 dy_1 \\ B(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) &= \iint \mathbf{1} \left[\tilde{A}(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) \leq \underline{a} \right] \\ &\quad \times f(y_0, y_1, X(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)))) \\ &\quad \times X_1(y_0, y_1, s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s))) dy_0 dy_1 \end{aligned}$$

Now, equating total population supply to total housing supply:

$$h(\mathcal{Z}^U(s))L(\mathcal{Z}^U(s))\mathcal{Z}_s^U(s) = G(s, \mathcal{Z}^U(s), q(\mathcal{Z}^U(s)), \mathcal{Z}_s^U(s))$$

where recall that $h(z)$ is the density of cities with income z . Re-arranging,

$$\mathcal{Z}_s^U(s) = \frac{A(s, q(\mathcal{Z}^U(s)), \mathcal{Z}^U(s))}{h(\mathcal{Z}^U(s))L(\mathcal{Z}^U(s)) - B(s, q(\mathcal{Z}^U(s)), \mathcal{Z}^U(s))}$$

It is easier at this stage to write the system in terms of the inverse matching function for unconstrained individuals $\mathcal{S}^U(z)$ for the range of cities in which unconstrained individuals live. Using the Migration Euler equation again, we finally obtain a nonlinear system of coupled Ordinary Differential Equations (ODEs):

$$\begin{aligned} \mathcal{S}_z^U(z) &= \frac{h(z)L(z) - B(\mathcal{S}^U(z), Q(L(z)), z)}{A(\mathcal{S}^U(z), Q(L(z)), z)} \\ L_z(z) &= \frac{R}{\mathcal{S}^U(z)Q'(L(z))} \end{aligned}$$

where recall that house prices are given by $q(z) = Q(L(z))$. The boundary conditions of this system are $\mathcal{S}^U(\bar{z}) = \bar{s}$, and $\mathcal{S}^U(\hat{z})$ given by total population supply, as defined below. When $\underline{s} > 0$ and f is bounded, inspection of this system reveals that it is uniformly Lipschitz continuous. In addition, the solution, if it exists, must be bounded. Indeed, diverging \mathcal{S}^U or $L(z)$ are ruled out by our compact support assumptions and by the fact that house prices cannot exceed income which is bounded above. Thus, conditional on boundary conditions, standard results ensure existence and uniqueness of a global solution to this system.

Recall that we assumed that the matching function $\mathcal{Z}^U(s)$ was locally increasing. We now show that the matching function $\mathcal{Z}^U(s)$ cannot be decreasing. The ODE without assuming that the matching function is increasing would be $|\mathcal{S}_z^U(z)| = \frac{h(z)L(z) - B(\mathcal{S}(z), Q(L(z)), z)}{A(\mathcal{S}(z), Q(L(z)), z)}$. Then, if the matching function has negative slope negative at some z_0 , since the right-hand-side is of constant sign and the matching function $\mathcal{Z}^U(s)$ cannot be flat (otherwise we would have a mass point, ruled out through the price function), the matching function $\mathcal{S}(z)$ cannot have a zero and hence is decreasing everywhere. Thus, house prices are concave throughout the support (from the no-arbitrage condition). Then we have $q'(z) = \mathcal{S}(z)/R < \bar{s}/R$, and hence $q(z) < q(\mathcal{S}^U(\hat{z})) + \bar{s}z/R$. Substituting back into the budget constraint of the individuals with skill in $(\bar{s} - ds, \bar{s}]$, they would have an incentive to increase their city choice, since this would yield a higher return on housing. This violates the Second Order Condition for optimality, and hence cannot hold in equilibrium.

A.1.3 Equilibrium in cities with only constrained individuals

We now turn to cities in which only constrained individuals live. We will apply the exact same logic as in the case for cities with at least one unconstrained individuals. We first define notation that is the counterpart

of $\mathcal{S}^C(y_0, y_1, z)$, but makes explicit the dependence on all endogenous objects:

$$C(y_0, y_1, q(z), q'(z)) = \frac{q'(z)(y_1 + Ra)}{\beta[y_0 - \underline{a} - q(z)] - zq'(z)}$$

and notice that $\mathcal{S}^C(y_0, y_1, z) = C(y_0, y_1, q(z), q'(z))$ at the equilibrium house rent schedule.

Total population in location z must satisfy

$$\begin{aligned} h(z)L(z) &= \iint \mathbf{1} \left[\tilde{A}(y_0, y_1, C(y_0, y_1, q(z), q'(z))) \leq \underline{a} \right] \\ &\quad \times f(y_0, y_1, C(y_0, y_1, q(z), q'(z))) \times \frac{d[C(y_0, y_1, q(z), q'(z))]}{dz} \times dy_0 dy_1 \end{aligned}$$

We can compute

$$\frac{d[C(y_0, y_1, q(z), q'(z))]}{dz} = C_0(y_0, y_1, z, q(z), q'(z)) + C_1(y_0, y_1, z, q(z), q'(z)) \times q''(z)$$

where we define

$$\begin{aligned} C_0(y_0, y_1, z, q(z), q'(z)) &= \frac{(1 + \beta)[q'^2[y_1 + Ra]}{\{\beta[y_0 - \underline{a} - q(z)] - zq'^2\}} \\ C_1(y_0, y_1, z, q(z), q'(z)) &= \frac{y_1 + Ra}{\beta[y_0 - \underline{a} - q(z)] - zq'(z)} + \end{aligned}$$

and hence

$$h(z)L(z) = D(z, q(z), q'(z)) + E(z, q(z), q'(z)) \times q''(z)$$

where

$$\begin{aligned} D(z, q(z), q'(z)) &= \iint \mathbf{1} \left[\tilde{A}(y_0, y_1, C(y_0, y_1, q(z), q'(z))) \leq \underline{a} \right] \\ &\quad \times f(y_0, y_1, C(y_0, y_1, q(z), q'(z))) \times C_0(y_0, y_1, z, q(z), q'(z)) \times dy_0 dy_1 \\ E(z, q(z), q'(z)) &= \iint \mathbf{1} \left[\tilde{A}(y_0, y_1, C(y_0, y_1, q(z), q'(z))) \leq \underline{a} \right] \\ &\quad \times f(y_0, y_1, C(y_0, y_1, q(z), q'(z))) \times C_1(y_0, y_1, z, q(z), q'(z)) \times dy_0 dy_1 \end{aligned}$$

which defines the nonlinear second-order ODE:

$$q''(z) = \frac{h(z)Q^{-1}(q(z)) - D(z, q(z), q'(z))}{E(z, q(z), q'(z))}$$

with boundary conditions $q(\hat{z}^-) = q(\hat{z}^+)$, $q(z_{min}) = 0$ that pins down z_{min} . In addition, q' must be continuous at the limiting point, otherwise there would be scope for arbitrage: $q'(\hat{z}^-) = q'(\hat{z}^+) = R/\mathcal{S}^U(\hat{z})$. The same argument as before ensures existence and uniqueness of the global solution conditional on boundary

conditions. Finally, $\mathcal{S}^U(\hat{z})$ is determined by the requirement that $\int h(z)L(z)dz = 1$, total population. Thus, an equilibrium exists.

A.2 Proof of Lemma 3

Suppose that $\underline{s} = \bar{s} = s$. Unconstrained individuals are indifferent between any of the locations in which there is at least one unconstrained individual. Because constrained individuals always locate in worst cities than any unconstrained individual of the same skill and we have only one skill type, it must be that constrained individuals all locate below \hat{z} . In other words, there is perfect segregation.

In this case, for unconstrained individuals,

$$R = \frac{s}{q'(z)}$$

This implies that for all cities $z \geq \hat{z}$,

$$\frac{d[q'(z)]}{dR} = -\frac{s}{R^2} < 0 \quad (15)$$

By continuity, this result extends to the case in which $\bar{s} - \underline{s}$ is strictly positive but small enough.

A.3 Proof of Lemma 4

First, we need to specify the production technology of housing. Suppose housing is produced using the final good k as sole input, according to $H = xk^\theta$, where $\eta = \frac{1-\theta}{\theta}$ and $q_0 = \frac{1}{\theta x^{1/\theta}}$. Under perfect competition in the housing sector, this production function results in the house rent prices used in the competitive equilibrium. The planner's problem can then be split into two stages: (1) allocate individuals over space to maximize second period output net of discounted housing creation, and (2) redistribute output for consumption. So if the planner can produce more output net of housing production than the competitive equilibrium, he can achieve any utilitarian or Pareto improvements over the competitive equilibrium. The planner chooses the joint distribution of (s, z) , $g(s, z)$, to solve

$$\begin{aligned} & \max_{g,k} \int szg(s, z)dsdz - R \int k(z)dz \\ s.t. & \int g(s, z)dz = f(s) \\ & \int g(s, z)ds = xk(z)^\theta \\ & \int g(s, z)dsdz = 1 \end{aligned}$$

where f is the given marginal skill distribution. Note that the planner discounts house production at the market interest rate, since if it did not use second period output to pay for housing today, it could save those resources which would deliver a gross return of R tomorrow.

First, we re-write this in terms of the shadow price of land that would prevail in the planner's allocation. We have after some algebra

$$k(z) = \left(\frac{L(z)}{x} \right)^{\frac{1}{\theta}} = (\theta x)^{\frac{1}{1-\theta}} q(z)^{\frac{1}{1-\theta}}$$

and hence

$$\begin{aligned} & \max_{g,q} \int s z g(s, z) dz dz - R(\theta x)^{\frac{1}{1-\theta}} \int q(z)^{1+\frac{1}{\eta}} dz \\ \text{s.t.} \quad & \int g(s, z) dz = f(s) \\ & \int g(s, z) ds = q_0^{-1/\eta} q(z)^{1/\eta} \end{aligned}$$

By construction, the planner's solution must yield weakly higher output than the competitive equilibrium.

Now, *conditional on* a shadow housing price schedule $q(z)$, this is a standard optimal transport problem, and given the supermodularity of the surplus sz , the solution is perfect Positive Assortative Matching (PAM): there exists an increasing matching function $S(z)$ such that

$$\int_{S(z)}^{\bar{s}} f(x) dx = q_0^{-1/\eta} \int_z^{\bar{z}} q(z')^{1/\eta} dz'$$

i.e.

$$\begin{aligned} S(z) &= \bar{F}^{-1} \left(\int_z^{\bar{z}} q(z')^{1/\eta} dz' \right) \\ f(S(z))S'(z) &= Q_0^{-1/\eta} q(z)^{1/\eta} \end{aligned}$$

where $\bar{F}(s) = 1 - F(s)$ is the skill tail cdf. In addition, from Theorem 4.7 p. 39 in Galichon (2016), we know that the solution is unique.

Now, the planner also chooses q . Clearly the house rent schedule from the competitive equilibrium is in the planner's choice set. Yet, we know that conditional on the competitive equilibrium's house rent schedule, the unique maximizer of the planner's problem features perfect PAM. Since the competitive equilibrium delivers imperfect PAM (the positive mass of constrained individuals do not satisfy strict PAM), the planner's solution must yield strictly higher gross output than the competitive equilibrium given the same house rent schedule.

In addition, since the planner can always choose the same house rent schedule as the competitive equilibrium, and the sorting of individuals differ strictly between both cases, it must be that output net of housing

costs is strictly higher in the planner's solution. In sum, the planner's solution yields strictly higher gross and net output compared to the competitive equilibrium.

A.4 Proof of Lemma 5

The proof is structured in three steps.

1. Show that city income net of rents is a sufficient statistic to capture welfare losses from the policy
2. Show that city income net of rents declines for all unconstrained individuals below the announced skill threshold
3. Show that this implies that it declines also for constrained individuals below the same skill threshold.

A.4.1 Indirect utility

We first go back to the problem of the individual and define indirect utility. For the unconstrained, consumption is

$$c_0 = \frac{1}{1 + \beta} \left[y_0 - q^* + \frac{y_1 + z^* s}{R} \right]$$

$$c_1 = \beta R c_0$$

where we denote optimal choices with asterisks (*), and omit dependence on individual characteristics for notational simplicity. Indirect utility of unconstrained individuals is

$$V^U := \beta \log \beta R + (1 + \beta) \log c_0$$

$$= \log \frac{(\beta R)^\beta}{(1 + \beta)^{1 + \beta}} + (1 + \beta) \log \left(y_0 + \frac{y_1}{R} + \frac{s z^*}{R} - q^* \right)$$

For the constrained, consumption is

$$c_0 = y_0 - q^* - \underline{a}$$

$$c_1 = y_1 + z^* s + R \underline{a}$$

and their indirect utility is

$$V^C = \log (y_0 - q^* - \underline{a}) + \beta \log (y_1 + z^* s + R \underline{a})$$

Consider a small change in q (dq) and zs ($d(zs)$). Then indirect utility changes according to

$$dV^C = -\frac{1}{c_0} dq + \frac{c_1}{\beta} d(zs)$$

Therefore, using the financial Euler equation,

$$c_0 \times dV^C < d \left[\frac{z^*s}{R} - q^* \right]$$

Therefore, if the right-hand-side is negative for the policy change (even though the change may be large, we can integrate the inequality across a sequence of infinitesimal changes), constrained individuals lose. In sum, for both constrained and unconstrained individuals, a decline in $\frac{z^*s}{R} - q^*$ entails a decline in indirect utility.

A.4.2 Income net of rent for unconstrained individuals

Define net income before the policy change as

$$I(y_0, y_1, s) = \frac{sz^*(y_0, y_1, s)}{R} - q(z^*(y_0, y_1, s))$$

and net income after the policy change as

$$\bar{I}(s) = \frac{z_0s}{R} - \bar{q}_0$$

where \bar{q}_0 is unique the rent after the policy change. For unconstrained individuals, we simplify notation to

$$I(y_0, y_1, s) \equiv I^U(s) = \frac{sZ^U(s)}{R} - q(Z^U(s))$$

because location choice does not depend on (y_0, y_1) conditional upon being unconstrained. For them, net income is an increasing and convex function of skill s . Indeed, differentiating it w.r.t. s :

$$\frac{d}{ds} \left(\frac{sZ^U(s)}{R} - q(Z^U(s)) \right) = \frac{Z^U(s)}{R} + \left(\frac{s}{R} - q'(Z^U(s)) \right) \cdot Z_s^U(s) = \frac{Z^U(s)}{R} > 0$$

where the last equality comes from the migration Euler equation.

After the policy change, matching still holds (even though it is degenerate) and hence the same formula applies. In this case the slope calculated in the previous equation is constant in s , and takes the unique value z_0/R .

We now turn to the rent after the policy change, \bar{q}_0 . Using the assumption $\eta < 1$, we can easily make

comparisons:

$$\begin{aligned}
\bar{q}_0 &= q_0 L_0^\eta \\
&= q_0 E[L]^\eta && \text{(where } L \text{ is the equilibrium population before policy change)} \\
&> q_0 E[L^\eta] && \text{(Jensen's inequality on the concave function } L \mapsto L^\eta) \\
&= E[q] \\
&> q(E[z]) && \text{(Jensen's inequality on the convex function } z \mapsto q(z))
\end{aligned}$$

Now, define $s_1 < s_0$ such that $Z^U(s_1) = E[z] < z_0 = Z^U(s_0)$. For unconstrained individuals with $s_1 \leq s \leq s_0$, since $I_s^U(s) = Z^U(s) \in [E[z], z_0]$, we can integrate to obtain

$$\frac{E[z](s_0 - s_1)}{R} < I^U(s_0) - I^U(s_1) < \frac{z_0(s_0 - s_1)}{R}$$

Therefore,

$$\begin{aligned}
I^U(s_1) &> I^U(s_0) - \frac{z_0(s_0 - s_1)}{R} \\
&= \frac{s_1 z_0}{R} - q(E[z]) \\
&> \frac{s_1 z_0}{R} - \bar{q}_0 = \bar{I}(s_1)
\end{aligned}$$

Hence, we know that at skill s_1 , net income for unconstrained individuals pre-reform is above net income post-reform. In addition, the slope of net income is lower pre-reform for $s \leq s_1$: it is $Z^U(s) \leq E[z]$ pre-reform, compared to $z_0 > E[z]$ post-reform.

The convexity of $I^U(s)$ then implies that

$$I^U(s) > \bar{I}(s) \quad , \quad \forall s \leq s_1$$

i.e. that all unconstrained individuals with lower skill than s_1 lose net income from the reform. Since net income is a sufficient statistic for indirect utility, unconstrained individuals with $s \leq S^U(E[z])$ lose from the policy.

A.4.3 Constrained individuals.

We can repeat exactly the same argument as for unconstrained individuals. We simply need to allow for dependence on (y_0, y_1) and leverage the monotonicity property of Z^C in skill. Define $s_0(y_0, y_1) < s_1(y_0, y_1)$ such that $\mathcal{Z}^C(y_0, y_1, s_1(y_0, y_1)) = E[z] < z_0 = \mathcal{Z}^C(y_0, y_1, s_0(y_0, y_1))$. Then the argument carries through, holding (y_0, y_1) fixed: the range is now for all constrained individuals with skill in $[\underline{s}, \mathcal{S}^C(y_0, y_1, E[z])]$. Since $\mathcal{S}^C(y_0, y_1, z) > \mathcal{S}^U(z)$, the range of skills for which constrained individuals lose is larger.

B Appendix: Calibration

We calibrate our infinite horizon economy to an annual level with two income states $N = 2$ for CRRA utility $u(c) = \frac{c^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$. We choose the parameter values in Table 6.

Table 6: Calibration Parameters

Parameter	Notation	Value
<i>Preferences</i>		
Discount Factor	β	0.95
Intertemporal Elasticity of Substitution	σ	0.01
<i>Idiosyncratic Income</i>		
Skill	s	8.00
Low Income State	y_1	0.10
High Income State	y_2	0.50
Transition Probability From Low to High	Λ_{12}	0.75
Transition Probability From High to Low	Λ_{21}	0.05
<i>Financial Markets</i>		
Risk-Free Rate	R	1.04
Credit Constraint	\underline{a}	0.00
<i>Cities</i>		
Best City	\bar{z}	1.00
Worst City	\underline{z}	0.00
House Rents Slope	$q'(z)$	$1.71 + 21.35 \cdot z^5$
House Rents	$q(z)$	$\int_{\underline{z}}^z q'(x)dx$

Most of those values are standard. For instance, if we interpret the low income state y_1 as unemployment and the high income state y_2 as employment, we can compute the stationary unemployment rate in this economy through the invariant distribution of the Markov chain transition matrix Λ' . At our current values, we obtain a stationary unemployment rate of 6.25%.

Our value of the Intertemporal Elasticity of Substitution σ (IES) is towards the low end of the accepted range. Our model is not a theory of what the correct value for the IES should be, and a full structural estimation would target relevant moments to estimate that parameter within our context. Because our current calibration is for illustration purposes, we maintain a low value to incentivize individuals to use the

location asset to smooth consumption.

The value of skill we use is relatively large, $s = 8$. Given our house rent schedule and the equilibrium city choice, this implies that the idiosyncratic component of income y_t represents between 15% and 20% of total labor income $y_t + sz_t$ depending on where individuals are in the state space. City income sz_t thus represents between 80% and 85%. This reflects the large observed differences in wages across cities. The differences in location between the best city 0.78 and the lowest city 0.74 individuals locate in, imply an income change of 0.32, which is in between the low and high idiosyncratic income states.

Finally, our house rents schedule is constructed in such a way that unconstrained individuals of skill $s = 8$ locate towards 75% of the best available city, and are free to downgrade as much as they like. It also implies housing expenses of about one third of total labor income, consistent with its empirical counterpart reported in Davis and Ortalo-Magne (2011).

To solve the model numerically, we adapt the method of endogenous gridpoints of Carroll (2006).

C Appendix: Data Description and Robustness Exercises

C.1 Data Description and Sample Selection

Our main data source are the DADS Postes. The DADS Postes are administrative tax data from the French statistical institute (INSEE). The DADS Postes are matched employer-employee datasets with rich information on the universe of workers who receive taxable labor income in France. The structure of the data is as follows. One DADS Postes dataset is a 2-year panel with the universe of workers in France. Each worker is uniquely identified in that 2-year panel. For each worker, we use the following variables:

- the start and end day of all of her employment spells
- for each employment spell:
 - total net wage earnings
 - age and gender
 - municipality of residence
 - district (departement) of birth
 - their employer’s unique firm identifier
 - occupation and industry 4-digit codes

As is common with matched employer-employee datasets, because the underlying data is reported by employers, we have no information on individuals when they are not employed. Therefore, for simplicity,

we label any individual who is not in the dataset as "Unemployed", even though we acknowledge that the labor force participation margin may be active for some individuals.

The French territory is partitioned in about 100 districts ("Departements") and 36,000 municipalities ("Communes"). Districts are fairly large areas (median area is 8,763 km² and median population is 531,380 inhabitants), while municipalities are much smaller (median area is slightly above 10 km², and median population is 432 inhabitants). We also use a measure of house prices at the district-year level obtained from INSEE, to compute real wages.

Individual identifiers are reset in each of those 2-year panels, so that we cannot link individuals across datasets. However, the time coverage of the panels overlap. For instance, the first panel we use has years 2002 and 2003, and the second has years 2003 and 2004. For this reason, we apply all our data selection criteria separately to each of those datasets and merge them at the end. Because we observe start and end dates of each spell, we can make sure that a given spell for a given individual is not duplicated in our final sample. For each of the DADS Postes, we do the following:

- We first compute a measure of how desirable a municipality is. To do so, we compute log nominal daily wages for each individual and each municipality. We then compute the average log nominal daily wage in each municipality. We then rank mean log nominal daily wages across municipalities in each year, and compute the corresponding percentile for each municipality. This is our measure of z .
- Second, we restrict the sample of individuals and employment spells in the following way:
 - Because the data comes at the annual level, we first combine any employment spells that end in the last day of the first year with a corresponding employment spell that starts in the first day of the second year, with the same individual and firm identifier, as one employment spell.
 - Second, we remove minor employment spell to ensure that each individual has only one employment spell at any given point in time. To do so, we keep only the highest paying employment spell in any given month for each individual.
 - Third, we define Employment-to-Employment (EE) and Employment-to-Unemployment-to-Employment (EUE) transitions as follows. An EE transition happens if an individual switches firms with less than forty days non-employed (the start day of the second employment spell minus the end day of the first employment spell is less than forty). We define an EUE transition if an individual switches firms with more than forty days non-employed.
 - Fourth, we restrict our sample to individuals who have exactly one EE transition or one EUE transition. Within 2 years:
 - * 1.7% of individuals experience at least one EUE transition. Of those individuals, 98% experience exactly one EUE transition.
 - * 0.9% of individuals experience at least one EE transition. Of those individuals, 98% experience exactly one EE transition.

Thus, our restriction keeps the vast majority of individuals who experience a transition. At this stage, we obtain a sample of individuals who make exactly one EE transition or one EUE transition. We thus observe exactly two employment spells per individual.

- Fifth, we keep only individuals for which each employment spell exceeds three months.
- We then combine those samples for each of the years between 2002-2003 and 2006-2007. We do so because of a major classification revision in 2008 and more minor one in 2002. We end up with a sample of 6,965,851 transitions.

C.2 Appendix: Robustness: Fixed City Ranks

Here we present robustness exercises to our empirical results in Section 4. Table 7 shows that the results with yearly municipality rank and observed wages in Tables 1 and 3 almost do not change when computing city ranks at the beginning of the sample and holding them fixed throughout. Table 8 shows that the results with yearly municipality rank and residual wages in Table 4 are also very similar when computing city ranks at the beginning of the sample and holding them fixed throughout. Finally, Table 9 shows that the results are also very close to those in Table 5 when including all individuals and not only movers.

Table 7: Unemployment Spells and Location Decisions: Movers and Observed Wage Growth

Fixed City Rank. Movers Only. Observed Wages.								
Origin Wage Percentile (OWP)	0.089*** (0.007)	0.100*** (0.005)	0.103*** (0.005)	0.159*** (0.006)	0.144*** (0.014)	0.144*** (0.012)	0.171*** (0.011)	0.244*** (0.012)
Wage Growth Percentile (WGP)					0.098*** (0.008)	0.100*** (0.007)	0.107*** (0.007)	0.124*** (0.007)
OWP*WGP					0.002 (0.015)	0.020 (0.013)	0.011 (0.013)	-0.001 (0.012)
Constant	-0.046*** (0.009)				-0.122*** (0.011)			
<i>Fixed Effects</i>								
Municipality-Year		✓	✓	✓		✓	✓	✓
Age			✓	✓			✓	✓
Birthplace			✓	✓			✓	✓
Gender			✓	✓			✓	✓
2-Digit Origin Occupation				✓				✓
2-Digit Origin Industry				✓				✓
Obs.	1959234	1918576	1913556	1375330	1959271	1927016	1913727	1375444
R^2	0.005	0.063	0.065	0.080	0.012	0.069	0.073	0.089
W.- R^2		0.006	0.006	0.012		0.013	0.015	0.021

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

26,868 Origin Municipalities; 2003-2007.

Note: Number of observations differs between regressions without and with wage controls because of missing characteristics.

Table 8: Unemployment Spells and Location Decisions: Movers and Residual Wage Growth

Fixed City Rank. Movers Only.
Wages net of age, gender, occupation and industry fixed effects.

Origin Wage Percentile (OWP)	0.089*** (0.007)	0.100*** (0.005)	0.103*** (0.005)	0.159*** (0.006)	0.185*** (0.012)	0.187*** (0.013)	0.222*** (0.011)	0.290*** (0.015)
Wage Growth Percentile (WGP)					0.097*** (0.008)	0.102*** (0.009)	0.109*** (0.008)	0.139*** (0.010)
OWP*WGP					-0.083*** (0.012)	-0.073*** (0.011)	-0.076*** (0.011)	-0.054*** (0.012)
Constant	-0.046*** (0.009)				-0.123*** (0.011)			
<i>Fixed Effects</i>								
Municipality-Year		✓	✓	✓		✓	✓	✓
Age			✓	✓			✓	✓
Birthplace			✓	✓			✓	✓
Gender			✓	✓			✓	✓
2-Digit Origin Occupation				✓				✓
2-Digit Origin Industry				✓				✓
Obs.	1959234	1918576	1913556	1375330	885838	849863	847780	847721
R^2	0.005	0.063	0.065	0.080	0.011	0.087	0.091	0.099
W.- R^2		0.006	0.006	0.012		0.012	0.014	0.021

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

26,868 Origin Municipalities; 2003-2007.

Note: Number of observations differs between regressions without and with wage controls because of missing characteristics.

Table 9: Unemployment Spells and Location Decisions: All Individuals and Residual Wage Growth

Fixed City Rank. All Individuals.
Wages net of age, gender, occupation and industry fixed effects.

Origin Wage Percentile (OWP)	0.024*** (0.002)	0.027*** (0.002)	0.027*** (0.002)	0.045*** (0.002)	0.050*** (0.003)	0.050*** (0.003)	0.060*** (0.003)	0.081*** (0.003)
Wage Growth Percentile (WGP)					0.026*** (0.002)	0.027*** (0.002)	0.029*** (0.002)	0.037*** (0.002)
OWP*WGP					-0.019*** (0.002)	-0.017*** (0.002)	-0.016*** (0.003)	-0.007* (0.003)
Constant	-0.012*** (0.002)				-0.034*** (0.002)			
<i>Fixed Effects</i>								
Municipality-Year		✓	✓	✓		✓	✓	✓
Age			✓	✓			✓	✓
Birthplace			✓	✓			✓	✓
Gender			✓	✓			✓	✓
2-Digit Origin Occupation				✓				✓
2-Digit Origin Industry				✓				✓
Obs.	6909084	6909084	6890553	4900170	3029703	3007472	3000347	3000300
R^2	0.001	0.026	0.027	0.035	0.003	0.042	0.044	0.046
W.- R^2		0.002	0.002	0.003		0.003	0.004	0.006

Standard errors in parentheses, clustered at the departement level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

26,868 Origin Municipalities; 2003-2007.

Note: Number of observations differs between regressions without and with wage controls because of missing characteristics.