

# Volume, Volatility and Public News Announcements\*

Tim Bollerslev,<sup>†</sup> Jia Li,<sup>‡</sup> and Yuan Xue<sup>§</sup>

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## Abstract

We provide new empirical evidence for the way in which financial markets process information. Our results are based on high-frequency intraday data along with new econometric techniques for making inference on the relationship between trading intensity and spot volatility around public news announcements. Consistent with the predictions derived from a theoretical model in which investors agree to disagree, our estimates for the intraday volume-volatility elasticity around the most important news announcements are systematically below unity. Our elasticity estimates also decrease significantly with measures of disagreements in beliefs, economic uncertainty, and textual-based sentiment, further highlighting the key role played by differences-of-opinion.

**Keywords:** Differences-of-opinion; high-frequency data; jumps; macroeconomic news announcements; trading volume; stochastic volatility; economic uncertainty; textual sentiment.

**JEL classification:** C51, C52, G12.

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<sup>†</sup>Department of Economics, Duke University, Durham, NC 27708; NBER; CREATES; e-mail: boller@duke.edu.

<sup>‡</sup>Department of Economics, Duke University, Durham, NC 27708; e-mail: jl410@duke.edu.

<sup>§</sup>Department of Economics, Duke University, Durham, NC 27708; e-mail: yuan.xue@duke.edu.

# 1 Introduction

Trading volume and return volatility in financial markets typically, but not always, move in tandem. By studying the strength of this relationship around important public news announcements, we shed new light on the way in which financial markets process new information and the key role played by differences-of-opinion among investors. Our empirical investigations rely critically on the use of high-frequency intraday data and new econometric procedures explicitly designed to deal with complications that arise in the high-frequency data setting.

An extensive empirical literature has documented the existence of a strong contemporaneous relation between trading volume and volatility; see Karpoff (1987) for a survey of some of the earliest empirical evidence. The mixture-of-distributions hypothesis (MDH), originally developed by Clark (1973) and later extended by Tauchen and Pitts (1983) and Andersen (1996) among others, provides a possible explanation for this empirical relationship based on the idea of a common news arrival process driving both the magnitude of returns and trading volume. The MDH, however, remains silent about the underlying economic mechanisms that link the actual trades and price adjustments to the news.

Meanwhile, a variety of equilibrium-based economic models have also been developed to help understand how prices and volume respond to new information. This includes the rational-expectations type models of Kyle (1985) and Kim and Verrecchia (1991) among many others, in which investors agree on the interpretation of the news, but their information sets differ. Although this class of models is able to account for the on-average positive correlation between volume and volatility observed empirically, the models are unable to explain the occurrence of abnormally large trading volumes that occasionally occur together with returns close to zero. Instead, models that feature differences-of-opinion, including those by Harrison and Kreps (1978), Harris and Raviv (1993), Kandel and Pearson (1995), Scheinkman and Xiong (2003) and Banerjee and Kremer (2010) among others, in which investors agree to disagree, may help explain this oft observed empirical phenomenon. In the differences-of-opinion class of models, the investors' interpretation of the news and their updated valuations of the assets do not necessarily coincide, thus allowing for the possibility of relatively small equilibrium price changes accompanied by relatively large aggregate trading volumes.

Most of the existing empirical evidence related to the economic models discussed above, and

the volume-volatility relationship in particular, has been based on daily or coarser frequency data.<sup>1</sup> Meanwhile, another more recent strand of literature has emphasized the advantages of the use of high-frequency intraday data for more accurately identifying “jumps” and studying the way in which financial prices respond to public news announcements; see, for example, Andersen et al. (2003), Andersen et al. (2007), Lee and Mykland (2008) and Lee (2012).<sup>2</sup> This naturally suggests that by “zooming in” and analyzing how not only prices but also trading volume and volatility evolve around important news announcements, a deeper understanding of the economic mechanisms at work and the functioning of markets may be forthcoming.

Set against this background, we provide new empirical evidence on the volume-volatility relationship around various macroeconomic news announcements based on high-frequency one-minute data for the S&P 500 aggregate market portfolio. We begin by documenting the occurrence of large increases in trading volume intensity around Federal Open Market Committee (FOMC) meetings without accompanying large price jumps. As noted above, this presents a challenge for models in which investors rationally update their beliefs based on the same interpretation of the news, and instead points to the importance of models allowing for disagreements or, differences-of-opinion, among investors.

To help further explore this thesis and guide our more in-depth empirical investigations, we derive an explicit expression for the elasticity of expected trading volume with respect to price volatility within the Kandel and Pearson (1995) differences-of-opinion model. We purposely focus our analysis on the elasticity as it may be conveniently estimated with high-frequency data and, importantly, has a clear economic interpretation in terms of model primitives. In particular, we show theoretically that the volume-volatility elasticity is monotonically decreasing in a well-defined measure of relative disagreement. Moreover, the elasticity is generally below one and reaches its upper bound of unity only in the benchmark case without disagreement.

The theoretical model underlying these predictions is inevitably stylized, focussing exclusively on the impact of public news announcements. As such, the theory mainly speaks to the “abnormal” movements in volume and volatility observed around these news events. To identify the abnormal movements, and thus help mitigate the effects of other confounding forces, we rely on the “jumps” in the volume intensity and volatility around the news announcements. Our estimation of the jumps is based on the differences between the post- and pre-event levels of the instantaneous

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<sup>1</sup>One notable exception is Chaboud et al. (2008), who document large trading volume in the foreign exchange market in the minutes immediately before macroeconomic announcements, even when the announcements are in line with market expectations and the actual price changes are small.

<sup>2</sup>Related to this, Savor and Wilson (2013) also document higher average excess market returns on days with important macroeconomic news releases compared to non-announcement days.

volume intensity and volatility, which we recover nonparametrically using high-frequency data. Even though the differencing step used in identifying the jumps effectively removes low-frequency dynamics in the volatility and volume series (including daily and lower frequency trending behavior) that might otherwise confound the estimates, the jump estimates are still affected by the well-documented strong intraday periodic patterns that exist in both volume and volatility (for some of the earliest empirical evidence, see Wood et al., 1985; Jain and Joh, 1988). In an effort to remove this additional confounding influence we apply a second difference with respect to a control group of non-announcement days. The resulting “doubly-differenced” jump estimates in turn serve as our empirical analogues of the abnormal volume and volatility movements that we use in our regression-based analysis of the theoretical predictions.

In its basic form, our empirical regression strategy may be viewed as a Differences-in-Differences (DID) type estimator, as commonly used in empirical microeconomic studies; see, for example, Ashenfelter and Card (1985). However, our setup is distinctly different from conventional settings, and the usual justification for the use of DID regressions does not apply in the high-frequency data setting. Correspondingly, our new econometric procedures and the justification thereof entail two important distinctions. Firstly, to accommodate the strong dynamic dependencies in the volatility and volume intensity, we provide a rigorous theoretical justification based on a continuous-time infill asymptotic framework allowing for essentially unrestricted non-stationarity. Secondly, we provide an easy-to-implement local *i.i.d.* bootstrap method for conducting valid inference. By randomly resampling only locally in time (separately before and after each announcement), the method provides a simple solution to the issue of data heterogeneity, which otherwise presents a key complication for bootstrapping in the high-frequency data setting (see, e.g., Gonçalves and Meddahi, 2009).

Our actual empirical findings are closely in line with the theoretical predictions derived from the Kandel and Pearson (1995) model and the differences-of-opinion class of models more generally. In particular, we first document that the estimated volume-volatility elasticity around FOMC announcements is significantly below unity. This finding carries over to other important intraday public news announcements closely monitored by market participants. Interestingly, the volume-volatility elasticity estimates are lower for announcements that are released earlier in the monthly news cycle (see Andersen et al., 2003), such as the ISM Manufacturing Index and the Consumer Confidence Index, reflecting the importance of timing across the announcements and the effect of learning.

Going one step further, we show that the intraday volume-volatility elasticity around news

announcements decreases significantly in response to increases in measures of dispersions-in-beliefs (based on the survey of professional forecasters as in, e.g., Van Nieuwerburgh and Veldkamp, 2006) and economic uncertainty (based on the economic policy uncertainty index of Baker et al., 2015). This again corroborates our theoretical predictions and the key role played by differences-of-opinion. Our more detailed analysis of FOMC announcements, in which we employ an additional textual-based measure for the negative sentiment in the accompanying FOMC statements (based on the methodology of Loughran and McDonald, 2011), further underscores the time-varying nature of the high-frequency volume-volatility relationship and the way in which the market processes new information: when the textual sentiment in the FOMC statement is more negative, the relative disagreement among investors also tends to be higher, pushing down the volume-volatility elasticity.

The rest of the paper is organized as follows. Section 2 presents the basic economic arguments and theoretical model that guide our empirical investigations. Section 3 describes the high-frequency intraday data and news announcements used in our empirical analysis. To help further motivate and set the stage for our more detailed subsequent empirical investigations, Section 4 discusses some preliminary findings specifically related to FOMC announcements. Section 5 describes the new high-frequency inference procedures that we rely on for our more in-depth empirical analysis. Section 6 presents our main empirical findings based on the full set of news announcements, followed by our more detailed analysis of FOMC announcements. Section 7 concludes. Technical details concerning the new econometric inference procedure are provided in Appendix A. Appendix B contains further data descriptions. Additional empirical results and robustness checks are relegated to a (not-for-publication) supplemental appendix.

## 2 Theoretical motivation

We rely on the theoretical volume-volatility relations derived from the differences-of-opinion model of Kandel and Pearson (1995) to help guide our empirical investigations. We purposely focus on a simplified version of the model designed to highlight the specific features that we are after, and the volume-volatility elasticity around news arrivals in particular. To contrast the differences-of-opinion and rational-expectations types of models, we also briefly outline the corresponding empirical implications from a simplified version of the Kim and Verrecchia (1991) model. We begin by discussing the basic setup and assumptions.

## 2.1 New information and differences-of-opinion

Following Kandel and Pearson (1995), henceforth KP, we assume that a continuum of traders trade a risky asset and a risk-free asset in a competitive market. The random payoff of the risky asset, denoted  $\tilde{u}$ , is unknown to the traders. The risk-free rate is normalized to be zero. The traders' utility functions have constant absolute risk aversion with risk tolerance  $r$ . Each trader receives a noisy private signal about the payoff of the risky asset. Conditional on  $\tilde{u}$ , the private signals are independent and normally distributed. There are only two types of traders,  $i \in \{1, 2\}$ , with the proportion of type 1 traders denoted  $\alpha$ . The precision of type- $i$  trader's signal is  $s_i$ .

The public signal is  $\tilde{u} + \tilde{\varepsilon}$ , where the noise term  $\tilde{\varepsilon}$  is normally distributed. After observing the public signal, the traders update their beliefs about  $\tilde{u}$  and optimally re-balance their positions. The key feature of the KP model is that the two types of traders agree to disagree on how to interpret the public signal when updating their beliefs about the asset value: type  $i$  traders believe that  $\tilde{\varepsilon}$  is drawn from the  $N(\mu_i, h^{-1})$  distribution. Differences-of-opinion regarding the public signal is present among the traders when  $\mu_1 \neq \mu_2$ .

Following KP it is possible to show that in equilibrium<sup>3</sup>

$$\text{Volume} = |\beta_0 + \beta_1 \cdot \text{Price Change}|, \quad (2.1)$$

where

$$\beta_0 = r\alpha(1 - \alpha)h(\mu_1 - \mu_2), \quad \beta_1 = r\alpha(1 - \alpha)(s_1 - s_2). \quad (2.2)$$

We remark that the coefficient  $\beta_0$  is directly associated with the degree of differences-of-opinion (i.e.,  $\mu_1 - \mu_2$ ), whereas  $\beta_1$  depends on the dispersion in the precisions of prior beliefs. Both coefficients are increasing in the degree of risk tolerance.

By way of contrast, we also consider the rational-expectations model of Kim and Verrecchia (1991), in which rational traders agree on the interpretation of the public signal. In this setting, only if the equilibrium price changes in response to the public signal, will the optimal positions of the traders change. In particular, in parallel to the expression above it is possible to show that<sup>4</sup>

$$\text{Volume} = |\beta_1 \cdot \text{Price Change}|. \quad (2.3)$$

Even though the economic mechanism of Kim and Verrecchia (1991) differs from that of the KP model, the empirical implications from equation (2.3) are obviously nested in equation (2.1) with  $\beta_0 = 0$ , that is, the case without differences-of-opinion.

<sup>3</sup>See equation (5) in Kandel and Pearson (1995).

<sup>4</sup>See Proposition 2 of Kim and Verrecchia (1991).

## 2.2 Expected volume and volatility

The implication of the KP model for the relationship between price adjustment and trading volume in response to new information is succinctly summarized by equations (2.1) and (2.2). These equations, however, depict an exact functional relationship between (observed) random quantities. A weaker, but empirically more realistic, implication can be obtained by thinking of this equilibrium relationship as only holding “on average.” Moment conditions corresponding to the stochastic version (2.1) formally capture this idea.

Specifically, let  $m(\sigma)$  denote the expected volume as a function of the volatility  $\sigma$  (i.e., the standard deviation of price change). Assuming that the price changes are normally distributed with mean zero and standard deviation  $\sigma$ , it follows by direct integration of (2.1) that

$$m(\sigma) = \sqrt{\frac{2}{\pi}} |\beta_1| \sigma \exp\left(-\frac{\beta_0^2}{2\beta_1^2 \sigma^2}\right) + |\beta_0| \left(2\Phi\left(\frac{|\beta_0|}{|\beta_1| \sigma}\right) - 1\right), \quad (2.4)$$

where  $\Phi$  denotes the cumulative distribution function of the standard normal distribution.<sup>5</sup> The expected volume  $m(\sigma)$  depends on  $\sigma$  and the  $(\beta_0, \beta_1)$  coefficients in a somewhat complicated fashion. However, it is straightforward to show that  $m(\sigma)$  is increasing and convex in  $\sigma$ .

In order to gain further insight regarding this (expected) volume-volatility relationship, Figure 1 illustrates how the  $m(\sigma)$  function varies with the “disagreement coefficient”  $\beta_0$ . Locally, when the volatility  $\sigma$  is close to zero, the expected volume is positive if and only if the opinions of the traders differ (i.e.,  $\mu_1 \neq \mu_2$ ). Globally, as  $\beta_0$  increases (from the bottom to the top curves in the figure), the equilibrium relationship between the expected volume and volatility “flattens out.”

It is useful for our analysis below to further quantify this global feature in terms of the elasticity of  $m(\sigma)$  with respect to  $\sigma$ . We will denote this elasticity by  $\mathcal{E}$ , and refer to it as the volume-volatility elasticity for short. A straightforward calculation yields

$$\mathcal{E} \equiv \frac{\partial m(\sigma)/m(\sigma)}{\partial \sigma/\sigma} = \frac{1}{1 + \psi(\gamma/\sigma)}, \quad (2.5)$$

where

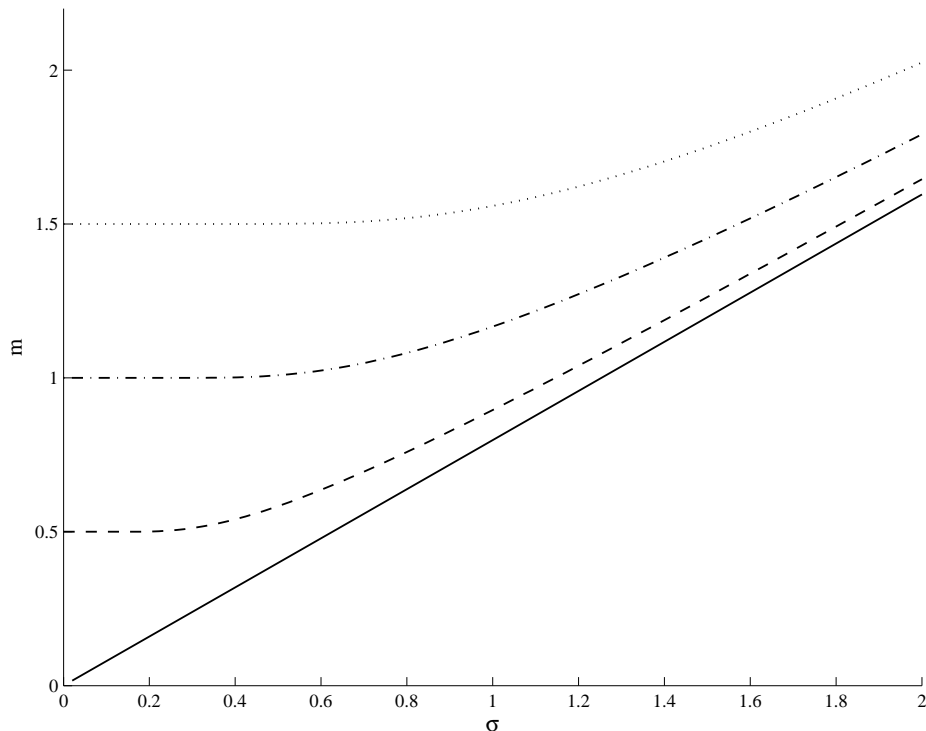
$$\gamma \equiv \frac{|\beta_0|}{|\beta_1|} = \frac{h |\mu_1 - \mu_2|}{|s_1 - s_2|}, \quad (2.6)$$

and the function  $\psi$  is defined by  $\psi(x) \equiv x(\Phi(x) - 1/2)/\phi(x)$ , with  $\phi$  being the density function of the standard normal distribution. The function  $\psi$  is strictly increasing on  $[0, \infty)$ , with  $\psi(0) = 0$  and  $\lim_{x \rightarrow \infty} \psi(x) = \infty$ .

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<sup>5</sup>Similarly, for the Kim and Verrecchia (1991) rational-expectations type model and the expression in (2.3),  $m(\sigma) = \sqrt{2} |\beta_1| \sigma / \sqrt{\pi}$ .

Figure 1: Equilibrium volume-volatility relations



*Notes:* The figure shows the equilibrium relationship between expected trading volume  $m$  and price volatility  $\sigma$  in the Kandel–Pearson model for various levels of disagreement, ranging from  $\beta_0 = 0$  (bottom) to 0.5, 1 and 1.5 (top).  $\beta_1$  is fixed at one in all of the graphs.

The expressions in (2.5) and (2.6) embody two important features of the volume-volatility elasticity that we use to guide our empirical analysis. Firstly,  $\mathcal{E} \leq 1$  with the equality and an elasticity of unity obtaining if and only if  $\gamma = 0$ . Secondly,  $\mathcal{E}$  only depends on and is decreasing in  $\gamma/\sigma$ . This second feature provides a clear economic interpretation of the volume-volatility elasticity  $\mathcal{E}$ : it is low when differences-in-opinion is *relatively* high, and vice versa, with  $\gamma/\sigma$  serving as the *relative* measure of the differences-of-opinion. This relative measure is higher when traders disagree more on how to interpret the public signal (i.e., larger  $|\mu_1 - \mu_2|$ ) and with more confidence (i.e., larger  $h$ ), relative to the degree of asymmetric private information (i.e.,  $|s_1 - s_2|$ ) and the overall price volatility (i.e.,  $\sigma$ ).

In our empirical investigations discussed below, we seek to directly quantify these relations based on intraday high-frequency transaction data around well-defined public news announcements, along with various proxies for the heterogeneity in beliefs and economic uncertainty associated with the news events. We turn next to a discussion of the data that we use in doing so.



Table 1: Summary statistics of high-frequency price returns and volume data

	Mean	Min	1%	25%	50%	75%	99%	Max
Return (percent)	0.000	-2.014	-0.149	-0.021	0.000	0.021	0.149	2.542
Volume (1,000 shares)	289.5	0.0	2.4	55.4	147.6	350.7	2005	31153

*Notes:* The table reports summary statistics of the one-minute returns and one-minute trading volumes for the SPY ETF during regular trading hours from April 10, 2001 to September 30, 2014.

### 3 Data description and summary statistics

Our empirical investigations are based on high-frequency intraday transaction prices and trading volume, together with precisely timed macroeconomic news announcements. We describe our data sets in turn.

#### 3.1 High-frequency market prices and trading volume

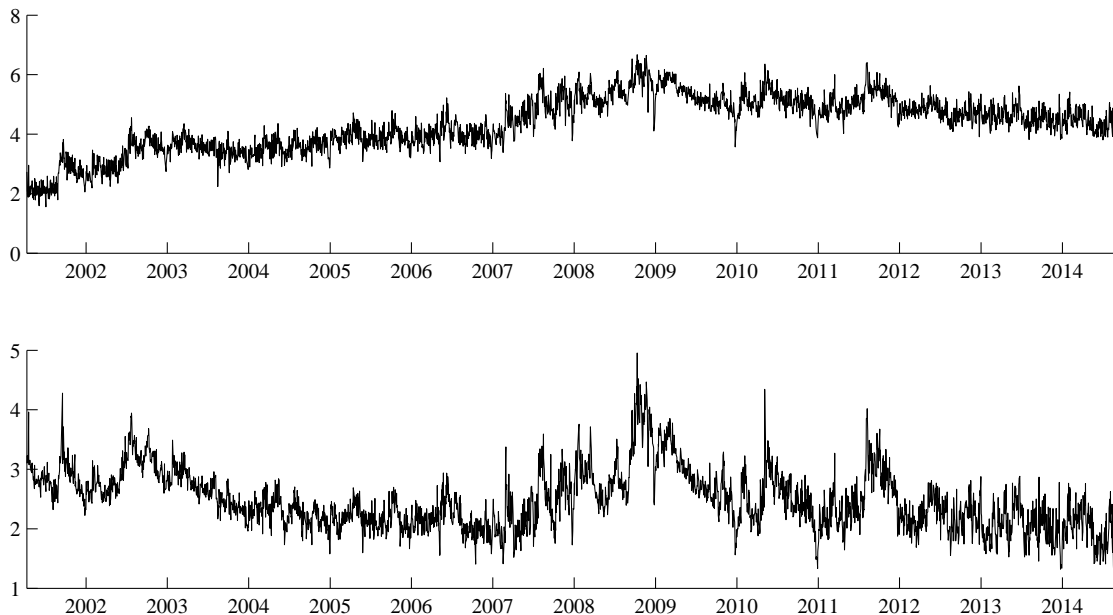
Our primary data is comprised of intraday transaction prices and trading volume for the S&P 500 index ETF (ticker: SPY). In some of our robustness checks we also employ high-frequency data of an ETF tracking the Dow Jones Industrial Average (ticker: DIA). All data are obtained from the TAQ database. The sample covers all regular trading days from April 10th, 2001 through September 30th, 2014. The raw data are cleaned and sampled at the one-minute frequency following the procedures detailed in Brownlees and Gallo (2006) and Barndorff-Nielsen et al. (2009). In total, there are 1,315,470 observations of one-minute return and trading volume.

Summary statistics for the SPY returns and trading volumes (number of shares) are reported in Table 1. Consistent with prior empirical evidence (see, e.g., Bollerslev and Todorov (2011)), the high-frequency one-minute returns appear close to be symmetrically distributed. The one-minute volume series, on the other hand, is highly skewed to the right, with occasionally very large values.

To highlight the general dynamic dependencies inherent in the data, Figure 2 plots the daily logarithmic trading volume (constructed by summing the one-minute trading volumes over each of the different days) and the logarithmic daily realized volatilities (constructed as the sum of squared one-minute returns over each of the days in the sample). As the figure shows, both of the daily series vary in a highly predictable fashion. The volume series, in particular, seems to exhibit an upward trend over the first half of the sample, but then levels off over the second half. Meanwhile, consistent with the extensive prior empirical evidence discussed above, there are strong dynamic commonalities evident in the two series.

In addition to the strong intertemporal dynamic dependencies, the volume and volatility series

Figure 2: Time series of volume and volatility



*Notes:* The figure shows the daily logarithmic trading volume (top panel) and logarithmic realized volatility (bottom panel) for the SPY ETF. The daily volume is constructed by accumulating the intraday volume. The daily realized volatility is constructed as the sum of one-minute squared returns over the day.

also exhibit strong intraday patterns. To illustrate this, Figure 3 plots the square-root of the one-minute squared returns averaged across each minute-of-the-day (as an estimate for the volatility over that particular minute) and the average trading volume over each corresponding minute. In order to prevent abnormally large returns and volumes from distorting the picture, we only include non-announcement days that are discussed in Section 3.2 below. Consistent with the evidence in the extant literature, there is a clear U-shaped pattern in the average volatility and trading activity over the active part of the trading day.<sup>6</sup>

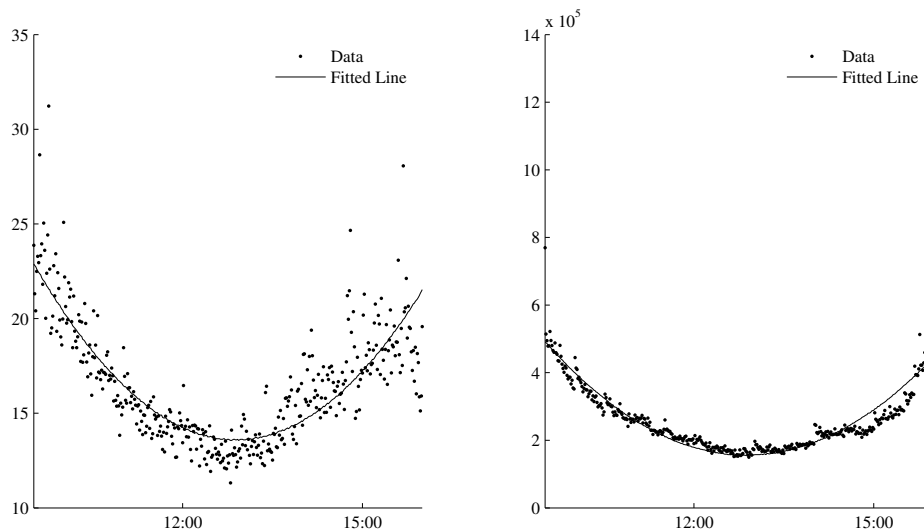
### 3.2 Macroeconomic news announcements

The Economic Calendar Economic Release section in Bloomberg includes the date and exact within day release time for over one-hundred regularly scheduled macroeconomic news announcements. Most of these announcements occur before the market opens or after it closes. We purposely focus on announcements that occur during regular trading hours only.<sup>7</sup> All-in-all, this leaves with 21

<sup>6</sup>See Wood et al. (1985), Harris (1986), Jain and Joh (1988), Baillie and Bollerslev (1990), and Andersen and Bollerslev (1997) for some of the earliest empirical evidence on the intraday patterns in volatility and volume.

<sup>7</sup>To ensure that there is a 30-minute pre-event (resp. post-event) window before (resp. after) each announcement, we exclude announcements that are released during the first and the last 30 minutes of the trading day.

Figure 3: Intraday patterns of volatility and volume



*Notes:* The figure shows the intraday volatility for the SPY ETF (left panel) constructed as the square root of the one-minute squared returns averaged across all non-announcement days, along with the best quadratic fit. The intraday trading volume for the SPY ETF (right panel) is similarly averaged across all non-announcement days.

different indicators for a total of 2,130 intraday public news announcements over the April 10, 2001 to September 30, 2014 sample period.

We identify four types of important announcements that comprise of the FOMC rate decision (FOMC), ISM Manufacturing Index (ISMM), ISM Non-Manufacturing Index (ISMNM), and the Consumer Confidence (CC) Index, based on prior empirical evidence.<sup>8</sup> Table 2 provides the typical release times and the number of releases over the sample for each of these important indicators. The remaining announcements are categorized as Others, a full list of which is provided in Table B.1 in Appendix B.

## 4 A preliminary analysis of FOMC announcements

To set the stage for our more in-depth subsequent empirical investigations, we begin by presenting a set of simple summary statistics and illustrative figures related to the volume-volatility relationship around FOMC announcements. We focus our preliminary analysis on FOMC announcements, because these are arguably among the most important public news announcements that occur during regular trading hours.<sup>9</sup>

<sup>8</sup>See, for example, Andersen et al. (2003), Boudt and Petitjean (2014), Jiang et al. (2011) and Lee (2012).

<sup>9</sup>The reaction of market prices to FOMC announcements has been extensively studied in the recent literature; see, for example, Johnson and Paye (2015) and many references therein.

Table 2: Macroeconomic news announcements

	No.Obs.	Time	Source
FOMC	109	14:15 <sup>†</sup>	Federal Reserve Board
ISM Manufacturing (ISMM)	160	10:00	Institute of Supply Management
ISM Non-Manufacturing (ISMNM)	158	10:00	Institute of Supply Management
Consumer Confidence (CC)	160	10:00	Conference Board
Other Indicators (Others)	1682	Varies	

*Notes:* The table reports the total number of observations, release time, and data source for each of the news announcements over the April 10, 2001 to September 30, 2014 sample.

<sup>†</sup>The exact times mostly vary from 14:00 to 14:15.

For each announcement, let  $\tau$  denote the pre-scheduled announcement time (typically at 14:15 EST). The time  $\tau$  is naturally associated with the integer  $i(\tau)$  such that  $\tau = (i(\tau) - 1)\Delta_n$ , where  $\Delta_n = 1$  minute is the sampling interval of our intraday data. We define the event window as  $((i(\tau) - 1)\Delta_n, i(\tau)\Delta_n]$ . Further, we define the pre-event (resp. post-event) window to be the  $k_n$ -minute period immediately before (resp. after) the event window. We denote the return and trading volume over the  $j$ th intraday time-interval  $((j - 1)\Delta_n, j\Delta_n]$  by  $r_j$  and  $V_{j\Delta_n}$ , respectively. The volume intensity  $m$  (i.e., the instantaneous mean volume) and the spot volatility  $\sigma$  before and after the announcement, denoted by  $m_{\tau-}$ ,  $m_\tau$ ,  $\sigma_{\tau-}$  and  $\sigma_\tau$ , respectively, may then be estimated by

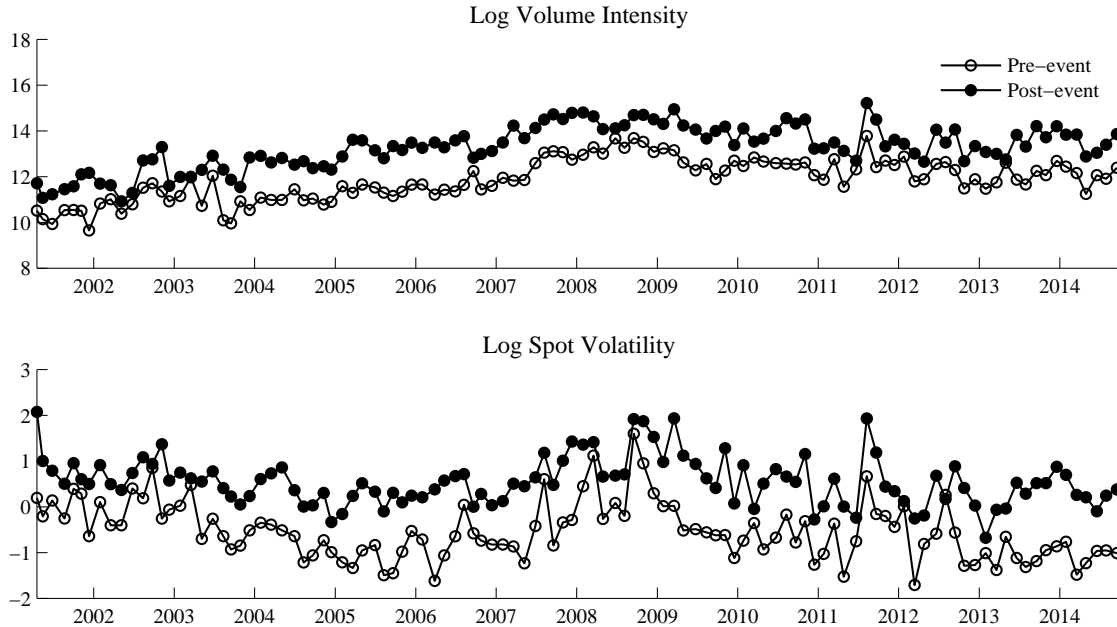
$$\begin{aligned}\hat{m}_{\tau-} &\equiv \frac{1}{k_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n}, & \hat{m}_\tau &\equiv \frac{1}{k_n} \sum_{j=1}^{k_n} V_{(i(\tau)+j)\Delta_n}, \\ \hat{\sigma}_{\tau-} &\equiv \sqrt{\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{i(\tau)-j}^2}, & \hat{\sigma}_\tau &\equiv \sqrt{\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{i(\tau)+j}^2}.\end{aligned}\tag{4.1}$$

These estimators are nonparametric, in the sense that they use data in local windows around the event time, where the window size  $k_n$  plays the role of the bandwidth parameter in usual nonparametric analysis. Under some standard technical assumptions that we detail in Appendix A, the validity of these nonparametric estimators can be justified. We set  $k_n = 30$  that corresponds to a 30-minute window.

Figure 4 plots the resulting time series of estimated logarithmic volume intensities (top panel) and logarithmic spot volatilities (bottom panel) before and after the FOMC announcements.<sup>10</sup> We observe marked bursts in the trading volume following each FOMC announcements, accompanied by positive jumps in the volatility. These jumps in the volume intensity and volatility are economically

<sup>10</sup>The logarithmic transform is naturally motivated by our interest in the volume-volatility elasticity derived in Section 2.2. The log-transformation also helps reduce the heteroskedasticity in both series. The heteroskedasticity in the spot volatility estimates is directly attributable to estimation errors. Formally, the standard error of the  $\hat{\sigma}_\tau$  estimate equals  $\hat{\sigma}_\tau/\sqrt{2k_n}$ , so that by the delta-method the standard error of  $\log(\hat{\sigma}_\tau)$  equals the constant  $1/\sqrt{2k_n}$ . In addition, the logarithm effectively transforms the salient multiplicative trend in the volume series over the first half of the sample to an additive trend, which as seen in Figure 2 is close to linear in time.

Figure 4: Volume and volatility around FOMC announcements



*Notes:* The figure plots the log-volume intensity (top panel) and log-volatility (bottom panel) around FOMC announcements. The volume intensity and volatility are calculated using equation (4.1) with  $k_n = 30$ .

large, with average jump sizes (in log) of 1.41 and 1.09, respectively.<sup>11</sup> This suggests that traders revise their beliefs about the stock market differently upon seeing the FOMC release. It is, of course, possible that traders have asymmetric private information about the overall market, although it seems much more likely that any differences are attributed to the traders’ different interpretation of the news.<sup>12</sup>

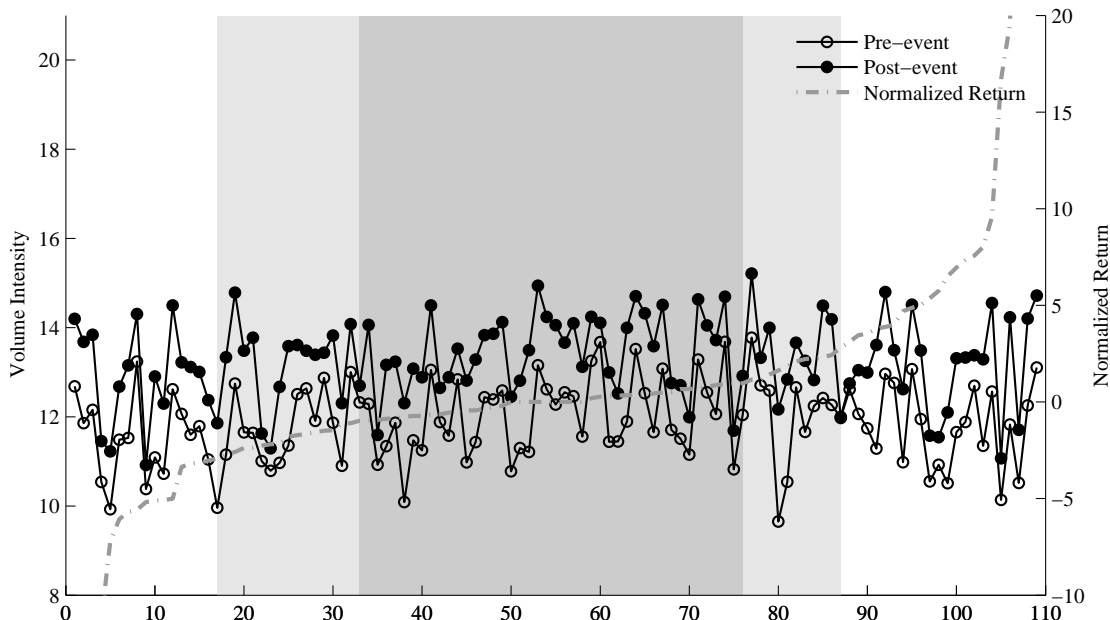
In order to further buttress the importance of differences-of-opinion among investors, we consider two additional empirical approaches. Firstly, as discussed in Section 2.1, if all investors agreed on the interpretation of the FOMC announcements, the trading volumes observed around the news releases should be approximately proportional to the price changes. Consequently, if there were no disagreement we would expect to see large price changes accompanied by large changes in trading volume and vice versa. To investigate this hypothesis, we sort all of the FOMC announcements by the normalized one-minute event returns  $r_i/\hat{\sigma}_{\tau-\sqrt{\Delta_n}}$ , and plot the resulting time series of pre- and post-event volume intensity estimates.<sup>13</sup> Consistent with the findings of KP (based on daily

<sup>11</sup>As discussed in Section 6, they are also highly statistically significant.

<sup>12</sup>It appears highly unlikely that there is any “insider information” pertaining to the actual FOMC release.

<sup>13</sup>The normalization with respect to the spot volatility serves as a scale adjustment to make the returns across announcements more comparable. A similar figure based on the five-minute returns is included in the supplemental

Figure 5: Sorted volume around FOMC announcements



*Notes:* The figure shows the pre- and post-event log volume intensities sorted on the basis of the 1-minute normalized returns  $r_{i(\tau)}/\hat{\sigma}_{\tau} - \sqrt{\Delta_n}$  around FOMC announcements (dots). The normalized return (dash-dotted line) increases from left to right. Announcements with normalized returns less than 1 (resp. 3) are highlighted by the dark (resp. light) shaded area.

data), Figure 5 shows no systematic association between trading volumes and returns. Instead, we observe many sizable jumps in the volume intensity for the absolute returns “close” to zero, that is, when they are less than three instantaneous standard deviations (light shaded area) or one instantaneous standard deviation (dark shaded area).

The empirical approach above mainly focuses on events with price changes close to zero and, hence, is *local* in nature. Our second empirical approach seeks to exploit a more *global* feature of the differences-of-opinion type models, namely that the volume-volatility elasticity should be below unity. While this prediction was derived from the explicit solution for the KP model in equation (2.4), the underlying economic intuition holds more generally: differences-of-opinion provides an additional trading motive that is not tied to the traders’ average valuation of the asset.

In order to robustly examine this prediction for the volume-volatility elasticity, without relying on the specific functional form in (2.5), we adopt a less restrictive reduced-form estimation strategy. Further along those lines, we note that the models discussed in Section 2 are inevitably stylized in nature, abstracting from other factors that might affect actual trading volume (e.g., liquidity

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appendix.

or life-cycle trading, reduction in trading costs, advances in trading technology, to name but a few). As such, the theoretical predictions are more appropriately thought of as predictions about “abnormal” variations in the volume intensity and volatility. In the high-frequency data setting, abnormal movements around announcements conceptually translate into “jumps” of the variables of interest. Below, we denote the log volatility jump by  $\Delta \log(\sigma_\tau) \equiv \log(\sigma_\tau) - \log(\sigma_{\tau-})$  and define  $\Delta \log(m_\tau)$  similarly for the volume.

These considerations naturally suggest the following reduced-form specification for estimating the volume-volatility elasticity  $\mathcal{E}$ ,

$$\Delta \log(m_\tau) = \text{Intercept} + \mathcal{E} \cdot \Delta \log(\sigma_\tau). \quad (4.2)$$

The estimation is carried out via a two-step semiparametric procedure. In the first step, we non-parametrically estimate the jumps using

$$\widehat{\Delta \log m_\tau} \equiv \log(\widehat{m}_\tau) - \log(\widehat{m}_{\tau-}), \quad \widehat{\Delta \log \sigma_\tau} \equiv \log(\widehat{\sigma}_\tau) - \log(\widehat{\sigma}_{\tau-}). \quad (4.3)$$

In the second step, we regress  $\widehat{\Delta \log m_\tau}$  on  $\widehat{\Delta \log \sigma_\tau}$  with an intercept. In Section 5, below, we formally justify this two-step procedure and develop the rigorous inference tools for gauging the nonparametric estimation error (which is nonstandard).

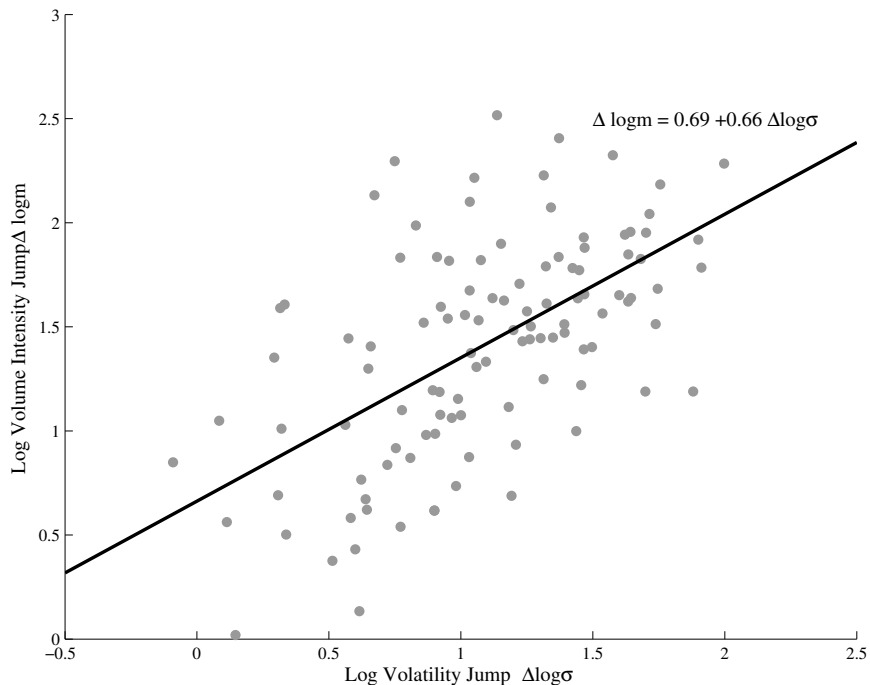
Figure 6 shows the corresponding scatter plot of the estimated FOMC log-volume intensity and log-volatility jumps. There is a clear positive correlation between the two series, with a correlation coefficient of 0.57. Moreover, the scatter plot does not reveal any obvious deviations from the simple log-linear relationship seen in equation (4.2). Consistent with the theoretical predictions, and the idea that traders interpret the FOMC announcements differently, the estimate of  $\widehat{\mathcal{E}} = 0.66$  is numerically less than unity.

The summary statistics and figures discussed above all corroborate the idea that differences-of-opinion among investors play an important role in the way in which the market responds to FOMC announcements. To proceed with a more formal empirical analysis involving other announcements and explanatory variables, we need econometric tools for conducting valid inference, to which we now turn.

## 5 High-frequency econometric procedures

The econometrics in the high-frequency setting is notably different from more conventional settings, necessitating the development of new econometric tools properly tailored to our empirical analysis

Figure 6: Volume and volatility jumps around FOMC announcements



*Notes:* The figure shows the scatter of the jumps in the log-volume intensity versus the jumps in the log-volatility around FOMC announcements. The line represents the least squares fit.

of the volume-volatility relationship.<sup>14</sup> To streamline the discussion, we focus on the practical implementation and heuristics for the underlying theory, deferring the technical details to Appendix A.

Our baseline econometric problem concerns the estimation and inference for the coefficients in a log-linear jump specification, like equation (4.2) in the preliminary descriptive analysis in the previous section. In addition to this simple specification for  $\mathcal{E}$ , we shall also investigate how other explanatory variables (e.g., types of announcements and measures of disagreement) might affect the volume-volatility elasticity. To do so, we parametrize both the intercept and the elasticity as linear functions of some explanatory variable  $X_\tau = (X_{0,\tau}, X_{1,\tau})$ , that is,

$$\Delta \log(m_\tau) = (a_0 + b_0^\top X_{0,\tau}) + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau). \quad (5.1)$$

This equation is best understood as an instantaneous moment condition, in which the volume intensity process  $m_t$  (resp. the spot volatility process  $\sigma_t$ ) represents the latent local first (resp. second) moment of the volume (resp. price return) process. Our goal is to conduct valid inference

<sup>14</sup>Aït-Sahalia and Jacod (2014) provide a comprehensive review of recent development on the econometrics of high-frequency data.



about the parameter  $\theta \equiv (a_0, b_0, a_1, b_1)$ , especially the components  $a_1$  and  $b_1$  that determine the volume-volatility elasticity.

Consider the group  $\mathcal{A}$  comprised of a total of  $M$  announcement times.<sup>15</sup> Further, define  $S_\tau \equiv (m_{\tau-}, m_\tau, \sigma_{\tau-}, \sigma_\tau, X_\tau)$  and  $\mathbf{S} \equiv (S_\tau)_{\tau \in \mathcal{A}}$ , where the latter collects the information on all announcements. Our estimator of  $\mathbf{S}$  may then be expressed as  $\widehat{\mathbf{S}}_n \equiv (\widehat{S}_\tau)_{\tau \in \mathcal{A}}$ , where  $\widehat{S}_\tau \equiv (\widehat{m}_{\tau-}, \widehat{m}_\tau, \widehat{\sigma}_{\tau-}, \widehat{\sigma}_\tau, X_\tau)$  is formed using the nonparametric pre- and post-event volume intensity and volatility estimators previously defined in (4.1). Correspondingly, summary statistics pertaining to the jumps in the volume intensity and volatility for the group of announcement times  $\mathcal{A}$  may be succinctly expressed as  $f(\widehat{\mathbf{S}})$  for some smooth function  $f(\cdot)$ .<sup>16</sup>

Moreover, we may estimate the parameter vector  $\theta \equiv (a_0, b_0, a_1, b_1)$  in (5.1) for the group  $\mathcal{A}$  using the following minimum-distance estimator

$$\widehat{\theta} \equiv \underset{\theta}{\operatorname{argmin}} \sum_{\tau \in \mathcal{A}} \left( \Delta \widehat{\log}(m_\tau) - (a_0 + b_0^\top X_{0,\tau}) - (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \widehat{\log}(\sigma_\tau) \right)^2. \quad (5.2)$$

This estimator may similarly be expressed as  $\widehat{\theta} = f(\widehat{\mathbf{S}})$ , albeit for a more complicated transform  $f(\cdot)$ . It can be shown that  $\widehat{\mathbf{S}}$  is a consistent estimator of  $\mathbf{S}$ , which in turn implies that  $f(\widehat{\mathbf{S}})$  consistently estimates  $f(\mathbf{S})$ , provided  $f(\cdot)$  is a smooth function of the estimated quantities. The estimates that we reported in our preliminary analysis in Section 4 may be formally justified this way.

The “raw” estimator defined above remains asymptotically valid under general technical conditions. However, the nonparametric estimators  $\Delta \widehat{\log}(\sigma_\tau)$  and  $\Delta \widehat{\log}(m_\tau)$  underlying the simple estimator in (5.2) do not take into account the strong intraday U-shaped patterns in trading volume and volatility documented in Figure 3. While the influence of the intraday patterns vanishes asymptotically, they invariably contaminate our estimates of the jumps in finite samples, and our use of the jump estimates as measures of “abnormal” volume and volatility movements, to which the economic theory speaks. A failure to adjust for this may therefore result in a mismatch between the empirical strategy and the economic theory.<sup>17</sup>

<sup>15</sup>Importantly, unlike conventional econometric settings, our asymptotic inference does not rely on an increasingly large number of announcements. Indeed, we assume that the sample span and the number of announcements within the span (i.e.,  $M$ ) are fixed. Our econometric theory exploits the fact that the high-frequency data are sampled at (asymptotically increasingly) short intervals. Our econometric setting allows for essentially arbitrary heterogeneity across the announcements and empirically realistic strong persistence in the volume intensity and volatility processes.

<sup>16</sup>For example, the average jump sizes in the logarithmic volatility and volume intensity around the announcements are naturally measured by

$$f(\widehat{\mathbf{S}}) = \frac{1}{M} \sum_{\tau \in \mathcal{A}} \Delta \widehat{\log}(\sigma_\tau) \quad \text{and} \quad f(\widehat{\mathbf{S}}) = \frac{1}{M} \sum_{\tau \in \mathcal{A}} \Delta \widehat{\log}(m_\tau),$$

respectively.

<sup>17</sup>For instance, the ISM indices and the Consumer Confidence Index are all released at 10:00 when volume and

To remedy this, we correct for the influence of the intraday pattern by differencing it out with respect to a control group. Since this differencing step is applied to the jumps, which are themselves differences between the post- and pre-event quantities, our empirical strategy may be naturally thought of as a high-frequency DID type estimator, in which we consider the event-control difference of the jump estimates as our measure for the abnormal movements in the volume intensity and volatility.

Formally, with each announcement time  $\tau$ , we associate a control group  $\mathcal{C}(\tau)$  of non-announcement times. Based on this control group, we then correct for the intraday patterns in the “raw” jump estimators by differencing out the corresponding estimates averaged within the control group, resulting in the adjusted jump estimators

$$\begin{aligned}\widetilde{\Delta \log(m_\tau)} &\equiv \Delta \widehat{\log(m_\tau)} - \frac{1}{N_C} \sum_{\tau' \in \mathcal{C}(\tau)} \Delta \widehat{\log(m_{\tau'})}, \\ \widetilde{\Delta \log(\sigma_\tau)} &\equiv \Delta \widehat{\log(\sigma_\tau)} - \frac{1}{N_C} \sum_{\tau' \in \mathcal{C}(\tau)} \Delta \widehat{\log(\sigma_{\tau'})},\end{aligned}\tag{5.3}$$

where  $N_C$  refers to the number of times in the control group.<sup>18</sup> The DID counterpart to (5.2) is then simply defined by,

$$\tilde{\theta} \equiv \underset{\theta}{\operatorname{argmin}} \sum_{\tau \in \mathcal{A}} \left( \widetilde{\Delta \log(m_\tau)} - (a_0 + b_0^\top X_{0,\tau}) - (a_1 + b_1^\top X_{1,\tau}) \cdot \widetilde{\Delta \log(\sigma_\tau)} \right)^2.\tag{5.4}$$

Note that  $\tilde{\theta}$  depends not only on  $(\widehat{S}_\tau)_{\tau \in \mathcal{A}}$  but also on  $(\widehat{S}_{\tau'})_{\tau' \in \mathcal{C}}$ , where  $\mathcal{C} \equiv \cup_{\tau \in \mathcal{A}} \mathcal{C}(\tau)$  contains the times of all control groups. This estimator can be expressed as  $\tilde{\theta} = f(\tilde{\mathbf{S}})$  where  $\tilde{\mathbf{S}} \equiv (\widehat{S}_\tau)_{\tau \in \mathcal{T}}$  for  $\mathcal{T} \equiv \mathcal{A} \cup \mathcal{C}$ .

In practice,  $\tilde{\theta}$  can easily be computed via an ordinary least square regression. However, the econometric inference (including the computation of standard errors) is non-standard. The sampling variability in  $\tilde{\theta}$  arises exclusively from the nonparametric estimation errors in the pre- and post-event high-frequency-based volume intensity and volatility estimators,  $\hat{m}_{\tau\pm}$  and  $\hat{\sigma}_\pm$ . While in theory it would be possible to characterize the resulting asymptotic covariance matrix and use it to design “plug-in” type standard errors, the control groups  $\mathcal{C}(\tau)$  for the different announcement times often partially overlap, which severely complicates the formal derivation and implementation of the requisite formulas.

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volatility both tend to be decreasing even on non-announcement days, while FOMC announcements mostly occur at 14:15 when volume and volatility are generally increasing.

<sup>18</sup>In our empirical analysis below,  $\mathcal{C}(\tau)$  consists of the same time-of-day as  $\tau$  over the previous  $N_C = 22$  non-announcement days (roughly corresponding to the length of one trading month). We also experimented with the use of other control periods, including periods comprised of future non-event days, resulting in the same general conclusions as the DID results reported below.

Instead, in order to facilitate the practical implementation, we rely on a novel easy-to-implement local *i.i.d.* bootstrap procedure for computing the standard errors. This procedure does not require the exact dependence of  $\tilde{\theta}$  on  $\tilde{\mathbf{S}}$  to be fully specified. Instead, it merely requires repeated estimation over a large number of locally *i.i.d.* bootstrap samples for the pre-event and post-event windows around each of the announcement and control times. The “localization” is important, as it allows us to treat the conditional distributions as (nearly) constant, in turn permitting the use of an *i.i.d.* re-sampling scheme.

The actual procedure is summarized by the following algorithm. The formal theoretical justification is given in the technical Appendix A.

### Bootstrap Algorithm

Step 1: For each  $\tau \in \mathcal{T}$ , generate *i.i.d.* draws  $(V_{i(\tau)-j}^*, r_{i(\tau)-j}^*)_{1 \leq j \leq k_n}$  and  $(V_{i(\tau)+j}^*, r_{i(\tau)+j}^*)_{1 \leq j \leq k_n}$  from  $(V_{i(\tau)-j}, r_{i(\tau)-j})_{1 \leq j \leq k_n}$  and  $(V_{i(\tau)+j}, r_{i(\tau)+j})_{1 \leq j \leq k_n}$ , respectively.

Step 2: Compute  $\Delta \log(m_\tau)^*$  and  $\Delta \log(\sigma_\tau)^*$  the same way as  $\Delta \log(m_\tau)$  and  $\Delta \log(\sigma_\tau)$ , respectively, except that the original data  $(V_{i(\tau)-j}, r_{i(\tau)-j})_{1 \leq |j| \leq k_n}$  is replaced with  $(V_{i(\tau)-j}^*, r_{i(\tau)-j}^*)_{1 \leq |j| \leq k_n}$ . Similarly, compute  $\tilde{\theta}^*$  according to (5.4) using re-sampled data.

Step 3: Repeat steps 1 and 2 a large number of times. Report the empirical standard errors of (the components of)  $\tilde{\theta}^* - \tilde{\theta}$  as the standard errors of the original estimator  $\tilde{\theta}$ .  $\square$

Equipped with the new high-frequency DID estimator defined in equation (5.4) and the accompanying bootstrap procedure outlined above for calculating standard errors and conducting valid inference, we now turn to our main empirical findings.

## 6 Volume-volatility relationship around public announcements

We begin our empirical investigations by verifying the occurrence of (on average) positive jumps in both trading volume intensity and return volatility around scheduled macroeconomic announcements. We document how these jumps, and the volume-volatility elasticity in particular, vary across different types of announcements. We then show how the variation in the elasticities observed across different announcements may be related to explanatory variables that serve as proxies for differences-of-opinion and, relatedly, notion of economic uncertainty. A more detailed analysis of FOMC announcements further highlights the important role played by the sentiment embedded in the FOMC statements accompanying each of the rate decisions.

Table 3: Volume and volatility jumps around public news announcements

Events	All	FOMC	ISM	ISMNM	CC	Others
No.Obs.	2130	109	160	158	160	1682
No DID						
Log Volatility	0.090** (0.004)	1.088** (0.027)	0.162** (0.017)	0.072** (0.018)	0.114** (0.015)	0.023** (0.005)
Log Volume	0.034** (0.004)	1.410** (0.023)	0.056** (0.015)	-0.045** (0.013)	0.049** (0.015)	-0.045** (0.005)
DID						
Log Volatility	0.152** (0.006)	1.037** (0.029)	0.256** (0.017)	0.165** (0.020)	0.209** (0.017)	0.087** (0.005)
Log Volume	0.204** (0.005)	1.329** (0.025)	0.335** (0.015)	0.233** (0.015)	0.328** (0.015)	0.118** (0.006)

*Notes:* The table reports the average logarithmic volatility jumps and the average logarithmic volume intensity jumps around all announcements (All), which are further categorized into FOMC announcements (FOMC), ISM Manufacturing Index (ISM), ISM Non-Manufacturing Index (ISMNM), Consumer Confidence Index (CC), and other pre-scheduled macroeconomic announcements (Others). The top panel reports the raw statistics. The bottom panel adjusts for the intraday pattern via the DID method using the past 22 non-announcement days as the control group. The sample period spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \*\* indicates significance at the 1% level.

## 6.1 Jumps and announcements

Consistent with the basic tenet of information-based trading around public news announcements, the preliminary analysis underlying Figure 4 clearly suggests an increase in both volatility and trading intensity from the thirty minutes before an FOMC announcement to the thirty minutes after the announcement. In order to more formally corroborate these empirical observations and extend them to a broader set of announcements, we report in Table 3 the average magnitudes of the logarithmic volatility and volume intensity jumps observed around news announcements, using the econometric inference procedure described in Section 5. We report the results for all of the news announcements combined, as well as the five specific news categories explicitly singled out in Table 2.

The top panel presents the “raw” jump statistics. The volatility jumps are always estimated to be positive and highly statistically significant. This is true for all of the announcements combined, as well as within each of the five separate categories. The volume jumps averaged across all news

announcements are also significantly positive. However, the jumps in the volume intensities are estimated to be *negative* for two of the news categories: ISM Non-Manufacturing and Others. This is difficult to reconcile with any of the economic mechanisms and theoretical models discussed in Section 2. Instead, these negative estimates may be directly attributed to the strong diurnal pattern evident in Figure 3. The ISM indices and most of the economic news included in the Others category are announced at 10:00am, when both volatility and trading volume tend to be falling, thus inducing a downward bias in the jump estimation.

To remedy this, we apply the DID estimation and inference approach discussed in the previous section, in which we rely on the previous 22 non-announcement days as the control group. As the resulting estimates reported in the bottom panel of Table 3 show, applying the DID correction results in significantly *positive* jumps for the spot volatility and trading intensity across *all* of the different news categories, ISMNM and Others included. This contrast directly underscores the importance of properly controlling for the intradaily features outside the stylized theoretical models when studying volume and volatility at the high-frequency intraday level. At the same time, the magnitude of the jump estimates associated with FOMC announcements, which mostly occur between 14:00 and 14:15 when volatility and trading volume both tend to be rising, is actually reduced by the DID correction. Nevertheless, FOMC clearly stands out among all of the different news categories, as having the largest (by a wide margin) average jump sizes in both volume and volatility.<sup>19</sup>

Having documented the existence of highly significant positive jumps in both volume and volatility around public announcements, we next turn to the joint relationship between the jumps, focussing on the volume-volatility elasticity and the implications of the theoretical models discussed in Section 2.

## 6.2 Volume-volatility elasticities around public news announcements

The theoretical models that guide our empirical investigations are explicitly designed to highlight how trading volume and return volatility respond to well-defined public news announcements. As such, the models are inevitably stylized, with other influences (such as those underlying the intraday patterns and long-term trends evident in Figures 3 and 4, respectively) deliberately abstracted away. As discussed above, the DID estimation approach provides a way to guard against the influence of the systematic intraday patterns. It also conveniently differences out other unmodeled nuisances, like trends, which would otherwise contaminate the estimates. Consequently, we rely on the DID estimation approach throughout.

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<sup>19</sup>We devote Section 6.4 to a more detailed separate investigation of FOMC announcements.

Table 4: Volume-volatility elasticities around public news announcements

	FOMC	ISMM	ISMNM	CC	Others
Constant ( $a_0$ )	0.586** (0.078)	0.199** (0.022)	0.119** (0.020)	0.218** (0.023)	0.050** (0.006)
Elasticity ( $a_1$ )	0.717** (0.067)	0.529** (0.064)	0.688** (0.069)	0.521** (0.075)	0.787** (0.022)
$R^2$	0.330	0.155	0.220	0.109	0.287

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \widehat{\log}(m_\tau) = a_0 - a_1 \cdot \Delta \widehat{\log}(\sigma_\tau)$ , using the past 22 non-announcement days as the control group. The sample spans the period from April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \*\* indicates significance at the 1% level.

To begin, consider a basic specification of equation (5.4) without any explanatory variables (i.e.,  $X_{0,\tau}$  and  $X_{1,\tau}$  are both absent). Table 4 reports the resulting estimates for each of the different news categories. All of the estimated intercepts (i.e.,  $a_0$ ) are positive and highly statistically significant, indicative of higher trading intensities following public news announcements, even in the absence of heightened return volatility.<sup>20</sup> Put differently, abnormal bursts in trading volume around announcements are not always associated with abnormal price changes. This, of course, is directly in line with the key idea underlying the KP model that differences-of-opinion provides an additional trading motive over explicit shifts in investors' average opinion. Further corroborating the rank of FOMC as the most important news category released during regular trading hours, the estimated intercept is the largest for FOMC announcements.

Turning to the volume-volatility elasticities (i.e.,  $a_1$ ), all of the estimates are below unity, and significantly so.<sup>21</sup> The theoretical derivations in (2.5) and (2.6) based on the KP model also predict that in the presence of differences-of-opinion the volume-volatility elasticity should be below unity. Our empirical findings are therefore directly in line with this theoretical prediction, and further support the idea that disagreements among investors often provide an important motive for trading.

### 6.3 Volume-volatility elasticities and disagreement measures

In addition to the prediction that the volume-volatility elasticity should be less than unity, our theoretical derivations in Section 2 also predict that the elasticity should be decreasing with the

<sup>20</sup>By contrast, the estimates obtained for  $a_0$  without the DID correction, reported in the supplementary appendix, are significantly negative for ISMM, ISMNM and the Others news categories, underscoring the importance of properly controlling for the strong intraday patterns in the volume intensity and volatility.

<sup>21</sup>The robust DID estimate for the elasticity around FOMC announcements reported in Table 4 is slightly larger than the preliminary raw non-DID estimate discussed in Section 4.

overall level of disagreement among investors. In order to examine this more refined theoretical prediction, we include a set of additional explanatory variables (in the form of the  $X_{1,\tau}$  variable in the specification in equation (5.4)) that serve as proxies for disagreement. To account for the category-specific heterogeneity in the volume-volatility elasticity estimates reported in Table 4, we also include a full set of category dummy variables (i.e., one for each of the FOMC, ISMM, ISMNM and CC news categories).

We consider two proxies for the overall level of investors' disagreement that prevails at the time of the announcement. The first is the forecast dispersion of the one-quarter-ahead unemployment rate from the Survey of Professional Forecasters (SPF).<sup>22</sup> This measure has also been used in previous studies to gauge the degree of disagreement; see, for example, Van Nieuwerburgh and Veldkamp (2006) and Ilut and Schneider (2014) among others. Secondly, as an indirect proxy for differences-of-opinion, we employ a weekly moving average of the economic policy uncertainty index developed by Baker et al. (2015).<sup>23</sup> There is a voluminous literature that addresses the relation between disagreement and uncertainty, generally supporting the notion of a positive relation between the two; see, for example, Acemoglu et al. (2006) and Patton and Timmermann (2010). Below, we refer to these two proxies as Dispersion and Weekly Policy, respectively. To facilitate comparisons, we scale both measures with their own sample standard deviations.

The estimation results for different specifications including these additional explanatory variables in the volume-volatility elasticity are reported in Table 5.<sup>24</sup> As a reference, the first column reports the results from a basic specification without any explanatory variables. The common elasticity is estimated to be 0.733 which, not surprisingly, is close to the average value of the category-specific estimates reported in Table 4. Underscoring the importance of disagreement more generally, the estimate is also significantly below one.

The specification in the second column includes the full set of news category dummies in the elasticity, with the baseline category being Others.<sup>25</sup> The elasticity for the Others category, which

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<sup>22</sup>The SPF is a quarterly survey. It is released and collected in the second month of each quarter. To prevent any look-ahead bias, we use the value from the previous quarter. Additional results for other forecast horizons and dispersion measures pertaining to other economic variables are reported in the supplemental appendix.

<sup>23</sup>The economic policy uncertainty index of Baker et al. (2015) is based on newspaper coverage frequency. We use the weekly moving average so as to reduce the noise in the daily index. The averaging also naturally addresses the weekly cycle in the media. Comparable results based on biweekly and monthly indices are available in the supplemental appendix.

<sup>24</sup>We also include news-category dummies in the intercept  $X_{0,\tau}$  in all of the different specifications, so as to control for the heterogeneity in the  $a_0$  estimates in Table 4. Since our main focus centers on the volume-volatility elasticity, to conserve space we do not report these estimated  $b_0$  dummy coefficients.

<sup>25</sup>Although the full set of news-category dummy variables are included in both the intercept and the elasticity specifications, the estimates in the second column in Table 5 are not exactly identical to those in Table 4, because some of the announcements across the different news categories occur concurrently.

Table 5: Volume-volatility elasticity estimates and disagreement measures

<i>Baseline estimates:</i>					
Constant ( $a_0$ )	0.044**	0.041**	0.040**	0.041**	0.041**
	(0.006)	(0.007)	(0.007)	(0.007)	(0.007)
Elasticity ( $a_1$ )	0.733**	0.776**	0.906**	0.921**	0.984**
	(0.020)	(0.024)	(0.043)	(0.036)	(0.045)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>					
<i>News-category dummy variables:</i>					
FOMC		-0.060	-0.058	-0.048	-0.049
		(0.072)	(0.072)	(0.072)	(0.072)
ISMM		-0.238**	-0.238**	-0.220**	-0.222**
		(0.070)	(0.070)	(0.070)	(0.070)
ISMNM		-0.090	-0.092	-0.082	-0.085
		(0.077)	(0.077)	(0.077)	(0.077)
CC		-0.244**	-0.244**	-0.202**	-0.207**
		(0.073)	(0.073)	(0.073)	(0.073)
<i>Disagreement measures:</i>					
Dispersion			-0.051**		-0.031*
			(0.013)		(0.014)
Weekly Policy				-0.079**	-0.070**
				(0.013)	(0.014)
$R^2$	0.481	0.482	0.483	0.486	0.486

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(\widetilde{m}_\tau) = a_0 + b_0^\top X_{0,\tau} + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\widetilde{\sigma}_\tau)$  based on all of the public announcements, using the past 22 non-announcement days as the control group. In all specifications,  $X_{0,\tau}$  include category dummy variables for FOMC rate decision (FOMC), ISM Manufacturing Index (ISMM), ISM Non-Manufacturing Index (ISMNM) and Consumer Confidence Index (CC); the estimates of these dummies (i.e.,  $b_0$ ) are not reported for brevity. The Dispersion variable is constructed as the latest forecast dispersion of the one-quarter-ahead unemployment rate from the Survey of Professional Forecasters before the announcement. The Weekly Policy variable is constructed as the weekly moving average before the announcement of the economic policy uncertainty index developed by Baker et al. (2015). Both variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.



includes by far the largest number of announcements, is estimated to be 0.776 and close to the value of 0.733 from the specification without any dummies. The estimates for FOMC and ISM Non-Manufacturing announcements are also both statistically indistinguishable from this value of 0.776. On the other hand, the volume-volatility elasticities estimated around ISM Manufacturing and Consumer Confidence announcements are both significantly lower, indicating that the levels of disagreement among investors are higher for these events. To help understand this latter finding, we note that ISM Manufacturing and Consumer Confidence announcements are both released early in the macroeconomic news cycle, as described in Andersen et al. (2003). As documented in Andersen et al. (2003, 2007), the first announcements in a given news cycle tend to have larger price impacts than later related news announcements, as much of the information contained in the later releases may have already been gleaned from the earlier news announcements. Our findings are in line with this logic. The estimated elasticities indicate relatively high levels of disagreement around the two early ISMM and CC announcements, and indirectly suggest that the release of these help resolve some of the economic uncertainty and decrease the overall level of disagreement, as manifest in the closer-to-one elasticity estimate for the later (in the news cycle) ISMNM releases.

The next two columns in the table report the results for specifications that include either Dispersion or Weekly Policy as an additional explanatory variable. Consistent with our theoretical prediction, both of these disagreement proxies significantly *negatively* impact the volume-volatility elasticity. Moreover, the estimated elasticities that obtain in the absence of any dispersion or economic policy uncertainty (i.e., the  $a_1$  coefficients) are much closer to the no-disagreement benchmark of unity implied by the theoretical expressions in (2.5) and (2.6), than the corresponding estimates obtained without controlling for disagreement.

The last column shows that both of the disagreement measures remain statistically significant when included jointly, although less so for Dispersion. Interestingly, the baseline elasticity of 0.984 for Others (i.e.,  $a_1$ ) is no longer statistically different from unity, nor are the estimated elasticities for FOMC and ISM Non-Manufacturing.

All-in-all, these results strongly corroborate the existence of a negative relationship between the volume-volatility elasticity and the level of disagreement among investors. The results also suggest that for a majority of the public news announcements, our two specific disagreement proxies, involving measures of forecast dispersion and economic policy uncertainty, are able to explain the deviation in the volume-volatility elasticity from the no-disagreement benchmark of unity.

## 6.4 Further analysis of FOMC announcements

The results discussed in the previous section were based on the joint estimation involving all of the macroeconomic news announcements that occur during regular trading hours. Meanwhile, as documented in Table 3, the FOMC rate decisions rank supreme in inducing the on-average largest jumps in both trading activity and return volatility over our sample. These large responses occur in spite of the fact that the federal funds rate was fixed at the effective zero lower bound over much of the later half of the sample. Moreover, economists also routinely disagree about the interpretation of monetary policy. All of these unique features grant FOMC announcements of particular interest for our analysis pertaining to the role of disagreement in financial markets.

Before diving into our more detailed empirical analysis of the volume-volatility elasticity estimated exclusively around the times of FOMC announcements, we want to stress some crucial differences between our analysis and prior work related to FOMC announcements. In particular, there is already an extensive literature devoted to the study of the impact of FOMC announcements on equity returns (e.g., Bernanke and Kuttner, 2005), including more recent studies specifically related to the behavior of monetary policy and market reactions when the rate is at or near the zero lower bound (e.g., Bernanke, 2012; Wright, 2012; van Dijk et al., 2014; Johnson and Paye, 2015). Other recent studies have also documented that most of the equity risk premium is earned in specific phases of the FOMC news release cycle (e.g., Savor and Wilson, 2014; Lucca and Moench, 2015; Cieslak et al., 2015). It is not our intent to add to this burgeoning literature on the determinant of the equity risk premium, and the functioning of monetary policy per se. Instead, we simply recognize the unique position of FOMC announcements as the most important news category in our sample. Motivated by this fact, we further investigate how the variation in the volume-volatility elasticity more generally varies with measures of disagreement prevailing at the exact time of and directly extracted from the FOMC news releases.

For ease of reference, the first column in Table 6 reports the DID estimation results for the FOMC subsample and the benchmark specification that does not include any explanatory variables in the elasticity, as previously reported in Table 4. The second column includes the previously defined Dispersion measure as an explanatory variable. In parallel to the full-sample results in Table 5, the estimates show that higher levels of forecast dispersions are generally associated with lower volume-volatility elasticities. The estimate of -0.108 for the  $b_1$  coefficient is also highly statistically significant. Moreover, after controlling for Dispersion, the baseline elasticity (i.e.,  $a_1$ ) is virtually one. This finding thus suggests that Dispersion alone, as a measure of differences-of-opinion, is able to successfully explain much of the deviation from unity in the volume-volatility

Table 6: Volume-volatility elasticity estimates around FOMC announcements

<i>Baseline estimates:</i>						
Constant ( $a_0$ )	0.586**	0.579**	0.583**	0.545**	0.550**	0.545**
	(0.078)	(0.079)	(0.079)	(0.078)	(0.079)	(0.079)
Elasticity ( $a_1$ )	0.716**	0.996**	0.790**	0.913**	1.089**	1.055**
	(0.067)	(0.085)	(0.082)	(0.082)	(0.092)	(0.093)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>						
Dispersion		-0.108**			-0.088**	-0.096**
		(0.019)			(0.020)	(0.021)
Weekly Policy			-0.037			0.046
			(0.021)			(0.028)
FOMC Sentiment				-0.104**	-0.077**	-0.097**
				(0.023)	(0.024)	(0.028)
$R^2$	0.330	0.382	0.329	0.370	0.400	0.400

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on FOMC announcements, using the past 22 non-announcement days as the control group. Dispersion and Weekly Policy are constructed as in Table 5. FOMC Sentiment is a textual measure constructed using financial-negative words in the FOMC press release. These variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

elasticity at the times of FOMC announcements.

The  $b_1$  estimate for the Weekly Policy variable reported in the third column of Table 6 is also negative. However, it is not significant at conventional levels. This lack of significance of the Weekly Policy variable in the FOMC subsample, stands in sharp contrast with its highly significant effect in Table 5 based on the full sample of all announcements. This therefore suggests that the Weekly Policy variable, which is constructed as a “catch-all” measure of economic uncertainty, is simply too diverse (or noisy) to satisfactorily explain the variation in the volume-volatility relationship observed exclusively around FOMC announcements.

To remedy this, we construct an alternative textual measure based on the actual FOMC press releases. The FOMC statements, in addition to announcing the new target rates, also outline

the longer-run goals of the Fed. In recent years, the statements also include brief summaries of the state of the economy, providing additional context underlying the rate decisions.<sup>26</sup> We succinctly summarize this additional information by counting the number of negative words, in accordance with the financial-negative (Fin-Neg) word list compiled by Loughran and McDonald (2011). We refer to this textual measure as the FOMC Sentiment. A more detailed description of the construction is provided in Appendix B.2.<sup>27</sup> Assuming that the use of more negative words provides additional room for investors to differ in their interpretation of the news, we consider this alternative FOMC Sentiment measure a more direct proxy for the level of disagreement at the exact times of the FOMC announcements.

From the theoretical relations derived in Section 2, we would therefore expect to see lower volume-volatility elasticities in response to higher FOMC Sentiment measures. The estimation results reported in the fourth column of Table 6 supports this theoretical prediction. The estimated  $b_1$  coefficient for our FOMC Sentiment measure equals -0.104. It is also highly statistically significant. Moreover, controlling for the FOMC Sentiment, the baseline elasticity (i.e.,  $a_1$ ) is estimated to be 0.914, and this estimate is statistically indistinguishable from unity at conventional significance levels. Interestingly, the  $b_1$  estimate of -0.104 for the FOMC Sentiment variable is also very close to the -0.108 estimate for the Dispersion measure reported in the second column.

In order to further gauge the relative merits of the Dispersion and FOMC Sentiment measures, the specification reported in the fifth column includes both as explanatory variables in the elasticity. Both of the estimated coefficients are negative and statistically significant. The coefficient estimates are also similar in magnitude, suggesting that the Dispersion and FOMC Sentiment measures are equally important in terms of capturing the disagreements-in-beliefs that motivate the abnormal trading at the times of FOMC announcements. Again, the estimate for the baseline elasticity  $a_1$  is also not statistically different from the theoretical prediction of unity that should obtain in the absence of differences-of-opinion.

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<sup>26</sup>This is especially important over the later half of our sample period, when the target rate was consistently stuck at the zero lower bound and, hence, offered little new information by itself. Bernanke (2012) also explicitly emphasized the important role of “public communications” as a nontraditional policy tool of the Fed.

<sup>27</sup>Loughran and McDonald (2011) originally constructed their Fin-Neg list for the purpose of analyzing corporate 10-K reports. Compared with negative words, positive words tend to be less informative due to their more frequent negation. Along those lines, some Fin-Neg words detected in the FOMC statement may not actually have a negative meaning. One example is the word “late” in the context “... are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014.” Another example is “unemployment,” which routinely appears in the first two paragraphs of the statements over the later half of our sample. To examine the severity of this issue, we manually checked every word that was classified as negative using Loughran and McDonald’s list, and then refined this selection by only keeping words with an unambiguous negative meaning within their context. The regression results based on this refined measure are very similar to those based on the FOMC Sentiment measure reported here, and hence are omitted for brevity.

Further augmenting the specification to include the Week Policy measure as an additional explanatory variable in the elasticity does not change the key aspects of any of these findings, as shown by the results reported in the last column of the table. Counter to the previous empirical results and theoretical predictions, the estimated coefficient for the Weekly Policy variable in this expanded DID regression is actually positive, albeit not significant at conventional levels. This therefore also indirectly supports the idea that our textual-based FOMC Sentiment measure provides a much more pointed and accurate characterization of the economic uncertainty and differences-in-opinion at the exact times of the FOMC announcements, compared to the “catch-all” Weekly Policy measure.

## 7 Conclusion

We provide new empirical evidence concerning the behavior of financial market volatility and trading activity in response to public news announcements. Our results are based on intraday prices and trading volume for the aggregate market portfolio, along with new econometric procedures specifically designed to deal with the unique complications that arise in the high-frequency data setting. Explicitly zooming in on the volume-volatility changes right around the exact times of the announcements allows us to cast new light on the way in which financial markets process new information and function more generally.

Consistent with the implications from theoretical models involving economic agents who agree-to-disagree, we find that the sensitivity of abnormal volume changes with respect to those of volatility estimated around the times of the most important public news announcements, as embedded within the volume-volatility elasticity, is systematically below unity. Further corroborating the important role played by differences-of-opinion among market participants, the elasticity tends to be low during times of high economic policy uncertainty and high dispersion among professional economic forecasters, and vice versa. A direct textual-based measure of the negative sentiment in the FOMC statements accompanying the actual rate decisions also negatively impacts the elasticity estimated at FOMC announcement times, lending additional empirical support to our key theoretical predictions.

## Appendix A: Technical background for econometric procedures

This appendix presents the formal econometric theory behind our high-frequency econometric estimation and inference procedures discussed in Section 5. Appendix A.1 describes the continuous-time setup for modeling the high-frequency price and volume data used in our empirical analysis. Appendix A.2 presents the main theoretical results, which we prove in Appendix A.3.

### A.1 Continuous-time setup and definitions

Throughout, we fix a filtered probability space  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ . Let  $(P_t)_{t \geq 0}$  denote the logarithmic price process of an asset. As is standard in the continuous-time finance literature (see, e.g., Merton (1992) and Duffie (2001)), we assume that  $P$  is a jump-diffusion process of the form

$$dP_t = b_t dt + \sigma_t dW_t + dJ_t, \tag{A.1}$$

where  $b$  is an instantaneous drift process,  $\sigma$  is a stochastic *spot volatility* process,  $W$  is a Brownian motion, and  $J$  is a pure jump process. The price is sampled at discrete times  $\{i\Delta_n : 0 \leq i \leq [T/\Delta_n]\}$ , where  $T$  denotes the sample span and  $\Delta_n$  denotes the sampling interval of the high-frequency data. We denote the corresponding high-frequency asset returns by  $r_i \equiv P_{i\Delta_n} - P_{(i-1)\Delta_n}$ .

Our empirical analysis is justified using an infill econometric theory with  $\Delta_n \rightarrow 0$  and  $T$  fixed. This setting is standard for analyzing high-frequency data (see, e.g., Ait-Sahalia and Jacod (2014) and Jacod and Protter (2012)) and it allows us to nonparametrically identify processes of interest in a general setting with essentially unrestricted nonstationarity and persistence.

We denote the trading volume within the high-frequency interval  $((i-1)\Delta_n, i\Delta_n]$  by  $V_{i\Delta_n}$ . Unlike the price, the high-frequency volume data cannot be realistically modeled using the jump-diffusion model. Following Li and Xiu (2016), we consider a general state-space model

$$V_{i\Delta_n} = \mathcal{V}(\zeta_{i\Delta_n}, \epsilon_{i\Delta_n}), \tag{A.2}$$

where  $\zeta$  is a latent state process,  $(\epsilon_{i\Delta_n})$  are i.i.d. transitory shocks with distribution  $F_\epsilon$ , and  $\mathcal{V}(\cdot)$  is a possibly unknown transform. The latent state process  $\zeta$  captures time-varying conditioning information such as the intensity of order arrival and the shape of the order size distribution. Technically, this state-space model can be formally defined on an *extension* of the space  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ ; see Li and Xiu (2016) for a formal construction.

This state-space model suits our empirical modeling of the trading volume well in two ways. Firstly, it conveniently allows the observed volume data to be discretely valued, while still being

completely flexible on the state space of the latent process  $\zeta$ . This allows us to model  $\zeta$  under minimal statistical restrictions without introducing unintended model inconsistency. Secondly, the setup does not restrict the dynamic persistence of the state process  $\zeta$ . This is important as volume data often exhibit nonstationary behavior.

Lastly, to complete the notation, let

$$m_t = \int \mathcal{V}(\zeta_t, \epsilon) F_\epsilon(d\epsilon) \quad (\text{A.3})$$

denote the instantaneous conditional mean process of  $V$ , that is, the *volume intensity* process. We use this process as the instantaneous empirical analogue to the expected volume in the theoretical models discussed in Section 2. Correspondingly, we use the spot volatility processes  $\sigma$  as the instantaneous analogue of the return standard deviation. Empirically, we should and do allow for general stochastic behavior in these instantaneous moments so as to accommodate essentially arbitrary stochastic behaviors of time-varying conditioning information.

## A.2 Estimation and inference

We now formally justify our inference procedures used in the main text. Below, we write  $\tilde{\theta}_n$  in place of  $\tilde{\theta}$  and similarly for other estimators, so as to emphasize the asymptotic stage. We focus on the DID estimator  $\tilde{\theta}_n$  defined by equation (5.4), which includes the “raw” estimator  $\hat{\theta}_n$  as a special case with the control group  $\mathcal{C}$  set to be empty. We write  $\tilde{\theta}_n = f(\tilde{\mathbf{S}}_n)$ , where  $f(\cdot)$  is defined implicitly by the definitions (5.3) and (5.4).

We assume the following regularity conditions for the underlying processes.

**Assumption 1.** (i) *The price process  $P$  is given by (A.1) for  $J_t = \int_0^t \xi_s dN_s + \int_0^t \int_{\mathbb{R}} \delta(s, z) \mu(ds, dz)$ , where the processes  $b$  and  $\sigma$  are càdlàg (i.e., right continuous with left limit) and adapted;  $\sigma_t$  is positive for  $t \in [0, T]$  almost surely; the process  $\xi$  is predictable and locally bounded;  $N$  is a counting process that jumps at the scheduled announcement times which are specified by the set  $\mathcal{A}$ ;  $\delta$  is a predictable function;  $\mu$  is a Poisson random measure with compensator  $\nu(ds, dz) = ds \otimes \lambda(dz)$  for some finite measure  $\lambda$ .*

(ii) *The volume process  $V$  satisfies (A.2). The process  $\zeta$  is càdlàg and adapted. The error terms  $(\epsilon_i)$  take values in some Polish space, are defined on an extension of  $(\Omega, \mathcal{F})$ , i.i.d. and independent of  $\mathcal{F}$ .*

(iii) *For a sequence of stopping times  $(T_m)_{m \geq 1}$  increasing to infinity and constants  $(K_m)_{m \geq 1}$ , we have  $\mathbb{E} |\sigma_{t \wedge T_m} - \sigma_{s \wedge T_m}|^2 + \mathbb{E} |\zeta_{t \wedge T_m} - \zeta_{s \wedge T_m}|^2 \leq K_m |t - s|$  for all  $t, s$  such that  $[s, t] \cap \mathcal{A} = \emptyset$ .*

Assumption 1 is fairly standard in the study of high-frequency data. Condition (i) allows the price process to contain jumps at both scheduled times and random times. Condition (ii) separates the conditional i.i.d. shocks  $(\epsilon_{i\Delta_n})$  at observation times from the latent continuous-time state process  $(\zeta_t)$ . This condition only mildly restricts the volume series, which can still exhibit essentially unrestricted conditional and unconditional heterogeneity through the (typically highly persistent) time-varying state process  $(\zeta_t)$ . Condition (iii) imposes a mild smoothness condition on  $\sigma$  and  $\zeta$  only in expectation, while allowing for general forms of jumps in their sample paths. This condition is satisfied for any semimartingales with absolutely continuous predictable characteristics (possibly with discontinuity points in  $\mathcal{A}$ ) and for long-memory type processes driven by the fractional Brownian motion.

In addition, we need the following conditions for the nonparametric analysis, where we denote  $M_p(\cdot) \equiv \int \mathcal{V}(\cdot, \epsilon)^p F_\epsilon(d\epsilon)$  for  $p \geq 1$ .

**Assumption 2.**  $k_n \rightarrow \infty$  and  $k_n^2 \Delta_n \rightarrow 0$ .

**Assumption 3.** (i) The function  $M_1(\cdot)$  is Lipschitz on compact sets and the functions  $M_2(\cdot)$  and  $M_4(\cdot)$  are continuous.

(ii) Almost surely, the function  $f$  is well-defined and continuously differentiable in a neighborhood of  $\tilde{\mathbf{S}}$ .

Assumption 2 specifies the growth rate of the local window size  $k_n$ . As typical in nonparametric analysis, this condition features a type of undersmoothing, so as to permit feasible inference. Assumption 3 imposes some smoothness conditions that are very mild.

We need some notations for stating the asymptotic results. For notational simplicity, we denote  $v_t = M_2(\zeta_t) - M_1^2(\zeta_t)$ . Consider variables  $(\eta_{\tau-}, \eta_\tau, \eta'_{\tau-}, \eta'_\tau)_{\tau \in \mathcal{T}}$  which, conditionally on  $\mathcal{F}$ , are mutually independent, centered Gaussian with variances  $(v_{\tau-}, v_\tau, \sigma_{\tau-}^2/2, \sigma_\tau^2/2)_{\tau \in \mathcal{T}}$ . We denote the first differential of  $f$  at  $\tilde{\mathbf{S}}$  with increment  $d\tilde{\mathbf{S}}$  by  $F(\tilde{\mathbf{S}}; d\tilde{\mathbf{S}})$ . For a sequence  $Y_n$  of random variables, we write  $Y_n \xrightarrow{\mathcal{L}\text{-}s} Y$  if  $Y_n$  converges stably in law towards  $Y$ , meaning that  $(Y_n, U)$  converges in distribution to  $(Y, U)$  for any bounded  $\mathcal{F}$ -measurable random variable  $U$ .

Theorem 1 characterizes the asymptotic distribution of the estimator  $\tilde{\theta}_n$ , and further shows that the asymptotic distribution can be consistently approximated using Algorithm 1.

**Theorem 1.** (a) Under Assumptions 1, 2 and 3

$$\sqrt{k_n}(\tilde{\theta}_n - \theta) \xrightarrow{\mathcal{L}\text{-}s} F(\tilde{\mathbf{S}}; (\eta_{\tau-}, \eta_\tau, \eta'_{\tau-}, \eta'_\tau, 0)_{\tau \in \mathcal{T}}). \quad (\text{A.4})$$

(b) Moreover, the conditional distribution function of  $\sqrt{k_n}(\tilde{\theta}_n^* - \tilde{\theta}_n)$  given the original data converges in probability to that of  $F(\tilde{\mathbf{S}}; (\eta_{\tau-}, \eta_\tau, \eta'_{\tau-}, \eta'_\tau, 0)_{\tau \in \mathcal{T}})$  under the uniform metric.



COMMENT. Since the variables  $(\eta_{\tau-}, \eta_{\tau}, \eta'_{\tau-}, \eta'_{\tau})_{\tau \in \mathcal{T}}$  are jointly  $\mathcal{F}$ -conditionally centered Gaussian, so is the limiting distribution given by (A.4).

### A.3 Proofs

Throughout the proofs, we use  $K$  to denote a generic constant which may change from line to line. For a generic random sequence  $Y_n$ , we write  $Y_n \xrightarrow{\mathcal{L}|\mathcal{F}} Y$  if the  $\mathcal{F}$ -conditional distribution function of  $Y_n$  converges in probability to that of  $Y$  under the uniform metric. By a classical localization procedure (see, e.g., Section 4.4.1 in Jacod and Protter (2012)), we can assume that the processes  $\sigma$  and  $\zeta$  are bounded, and piecewise (1/2)-Hölder continuous under the  $L^2$ -norm (with possible discontinuity points given by  $\mathcal{A}$ ) without loss of generality. We consider a sequence  $\Omega_n$  of events given by

$$\Omega_n \equiv \{ \text{the intervals } [\tau - 2k_n\Delta_n, \tau + 2k_n\Delta_n], \tau \in \mathcal{T}, \text{ are mutually disjoint} \}.$$

Since  $\mathcal{T}$  is finite,  $\mathbb{P}(\Omega_n) \rightarrow 1$ . Therefore, we can focus attention on  $\Omega_n$ , again without loss of generality.

**Proof of Theorem 1(a).** We first show that

$$\sqrt{k_n}(\hat{m}_{n,\tau-} - m_{\tau-}, \hat{m}_{n,\tau} - m_{\tau})_{\tau \in \mathcal{T}} \xrightarrow{\mathcal{L}|\mathcal{F}} (\eta_{\tau-}, \eta_{\tau})_{\tau \in \mathcal{T}}. \quad (\text{A.5})$$

In restriction to  $\Omega_n$ , the estimators on the left-hand side of (A.5) are  $\mathcal{F}$ -conditionally independent. Therefore, it suffices to establish the convergence for the marginal distributions. We thus focus on  $\sqrt{k_n}(\hat{m}_{n,\tau} - m_{\tau})$ , noting that the proof concerning  $\sqrt{k_n}(\hat{m}_{n,\tau-} - m_{\tau-})$  is similar.

We decompose  $\sqrt{k_n}(\hat{m}_{n,\tau} - m_{\tau}) = A_n + R_n$ , where

$$\begin{aligned} A_n &\equiv \frac{1}{\sqrt{k_n}} \sum_{j=1}^{k_n} (V_{(i(\tau)+j)\Delta_n} - M_1(\zeta_{(i(\tau)+j)\Delta_n})), \\ R_n &\equiv \frac{1}{\sqrt{k_n}} \sum_{j=1}^{k_n} (M_1(\zeta_{(i(\tau)+j)\Delta_n}) - M_1(\zeta_{\tau})). \end{aligned}$$

By Assumption 3,  $\mathbb{E}|R_n| \leq Kk_n\Delta_n^{1/2}$ , which goes to zero by Assumption 2.

It remains to consider the convergence of  $A_n$ . Since  $(V_{i\Delta_n})_{i \geq 0}$  are  $\mathcal{F}$ -conditionally independent,

$$\mathbb{E}[A_n^2|\mathcal{F}] = \frac{1}{k_n} \sum_{j=1}^{k_n} v_{(i(\tau)+j)\Delta_n} \rightarrow v_{\tau}, \quad (\text{A.6})$$

where the convergence holds because the process  $v$  is càdlàg. Moreover, by the continuity of  $M_4(\cdot)$  and the boundedness of  $\zeta$ , we can verify a Lyapunov-type condition:

$$\mathbb{E} \left[ \left| V_{(i(\tau)+j)\Delta_n} - M_1(\zeta_{(i(\tau)+j)\Delta_n}) \right|^4 \middle| \mathcal{F} \right] \leq K M_4(\zeta_{(i(\tau)+j)\Delta_n}) \leq K. \quad (\text{A.7})$$

By (A.6) and (A.7), we can apply the Lindeberg central limit theorem under the  $\mathcal{F}$ -conditional probability, resulting in  $\sqrt{k_n}(\hat{m}_\tau - m_\tau) \xrightarrow{\mathcal{L}|\mathcal{F}} \eta_\tau$  as claimed.

Since the jumps of  $P$  are of finite activity, the returns involved in  $\hat{\sigma}_{n,\tau-}$  and  $\hat{\sigma}_{n,\tau}$  do not contain jumps, with probability approaching one. By Theorem 13.3.3(c) of Jacod and Protter (2012), we have

$$\sqrt{k_n}(\hat{\sigma}_{n,\tau-} - \sigma_{n,\tau-}, \hat{\sigma}_{n,\tau} - \sigma_{n,\tau})_{\tau \in \mathcal{T}} \xrightarrow{\mathcal{L}-s} (\eta'_{\tau-}, \eta'_\tau)_{\tau \in \mathcal{T}}. \quad (\text{A.8})$$

By Proposition 5 of Barndorff-Nielsen et al. (2008) and the property of stable convergence in law, we can combine (A.5) and (A.8), yielding

$$\sqrt{k_n}(\tilde{\mathbf{S}}_n - \tilde{\mathbf{S}}) \xrightarrow{\mathcal{L}-s} (\eta_{\tau-}, \eta_\tau, \eta'_{\tau-}, \eta'_\tau, 0)_{\tau \in \mathcal{T}}. \quad (\text{A.9})$$

The assertion (A.4) then follows from (A.9) and the delta method. *Q.E.D.*

**Proof of Theorem 1(b).** Step 1. We divide the proof into several steps. Denote  $\mathcal{G} \equiv \mathcal{F} \vee \sigma\{\epsilon_i : i \geq 0\}$ . In this step, we show that, for each  $\tau \in \mathcal{T}$ ,

$$\sqrt{k_n}(\hat{m}_{n,\tau-}^* - \hat{m}_{\tau-}, \hat{\sigma}_{n,\tau-}^* - \hat{\sigma}_{\tau-}) \xrightarrow{\mathcal{L}|\mathcal{G}} (\eta_{\tau-}, \eta'_{\tau-}). \quad (\text{A.10})$$

Observe that

$$\sqrt{k_n} \begin{pmatrix} \hat{m}_{\tau-}^* - \hat{m}_{\tau-} \\ \hat{\sigma}_{\tau-}^{*2} - \hat{\sigma}_{\tau-}^2 \end{pmatrix} = \frac{1}{\sqrt{k_n}} \sum_{j=1}^{k_n} \left( \begin{pmatrix} V_{(i(\tau)-j)\Delta_n}^* \\ r_{i(\tau)-j}^{*2}/\Delta_n \end{pmatrix} - \frac{1}{k_n} \sum_{j=1}^{k_n} \begin{pmatrix} V_{(i(\tau)-j)\Delta_n} \\ r_{i(\tau)-j}^2/\Delta_n \end{pmatrix} \right). \quad (\text{A.11})$$

By the construction of the bootstrap sample, the summands in the right-hand side of (A.11) are i.i.d. with zero mean conditional on  $\mathcal{G}$ . We denote the  $\mathcal{G}$ -conditional covariance matrix of  $\sqrt{k_n}(\hat{m}_{\tau-}^* - \hat{m}_{\tau-}, \hat{\sigma}_{\tau-}^{*2} - \hat{\sigma}_{\tau-}^2)$  by

$$\Sigma_{n,\tau-} = \begin{pmatrix} \Sigma_{n,\tau-}^{(11)} & \Sigma_{n,\tau-}^{(12)} \\ \Sigma_{n,\tau-}^{(12)} & \Sigma_{n,\tau-}^{(22)} \end{pmatrix},$$

where

$$\begin{cases} \Sigma_{n,\tau-}^{(11)} = \frac{1}{k_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n}^2 - \left( \frac{1}{k_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n} \right)^2, \\ \Sigma_{n,\tau-}^{(12)} = \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n} r_{i(\tau)-j}^2 - \left( \frac{1}{k_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n} \right) \left( \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{i(\tau)-j}^2 \right), \\ \Sigma_{n,\tau-}^{(22)} = \frac{1}{k_n \Delta_n^2} \sum_{j=1}^{k_n} r_{i(\tau)-j}^4 - \left( \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{i(\tau)-j}^2 \right)^2. \end{cases}$$

In step 2, we shall show that

$$\left( \Sigma_{n,\tau-}^{(11)}, \Sigma_{n,\tau-}^{(12)}, \Sigma_{n,\tau-}^{(22)} \right) \xrightarrow{\mathbb{P}} (v_{\tau-}, 0, 2\sigma_{\tau-}^4). \quad (\text{A.12})$$

For each subsequence  $\mathbb{N}_1 \subseteq \mathbb{N}$ , we can find a further subsequence  $\mathbb{N}_2 \subseteq \mathbb{N}_1$  such that the convergence in (A.12) holds almost surely along  $\mathbb{N}_2$ . Moreover, on the paths for which this convergence holds, we apply a central limit theorem under the  $\mathcal{G}$ -conditional probability to deduce the convergence of the conditional law of  $\sqrt{k_n}(\hat{m}_{\tau-}^* - \hat{m}_{\tau-}, \hat{\sigma}_{\tau-}^* - \hat{\sigma}_{\tau-})$  towards that of  $(\eta_{\tau-}, \eta'_{\tau-})$  along the subsequence  $\mathbb{N}_2$ . From here, we deduce (A.10) for the original sequence by reversing the subsequence argument.

Step 2. We show (A.12) in this step. We start with  $\Sigma_{n,\tau-}^{(11)}$ . Consider the decomposition

$$\frac{1}{k_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n}^2 = A_{1,n} + A_{2,n},$$

where

$$\begin{aligned} A_{1,n} &\equiv \frac{1}{k_n} \sum_{j=1}^{k_n} \left( V_{(i(\tau)-j)\Delta_n}^2 - M_2(\zeta_{(i(\tau)-j)\Delta_n}) \right), \\ A_{2,n} &\equiv \frac{1}{k_n} \sum_{j=1}^{k_n} M_2(\zeta_{(i(\tau)-j)\Delta_n}). \end{aligned}$$

Note that the summands in  $A_{1,n}$  are  $\mathcal{F}$ -conditionally independent with zero mean. Therefore,

$$\mathbb{E} [A_{1,n}^2 | \mathcal{F}] \leq K k_n^{-2} \sum_{j=1}^{k_n} M_4(\zeta_{(i(\tau)-j)\Delta_n}) \leq K k_n^{-1} \rightarrow 0,$$

which further implies that  $A_{1,n} = o_p(1)$ . In addition, since  $M_2(\cdot)$  is continuous and  $\zeta$  is càdlàg,  $A_{2,n} \rightarrow M_2(\zeta_{\tau-})$ . From here and (A.9), it follows that

$$\Sigma_{n,\tau-}^{(11)} \xrightarrow{\mathbb{P}} M_2(\zeta_{\tau-}) - M_1(\zeta_{\tau-})^2 = v_{\tau-}.$$

Next, we consider the limiting behavior of  $\Sigma_{n,\tau-}^{(12)}$ . We decompose

$$\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} V_{(i(\tau)-j)\Delta_n} r_{i(\tau)-j}^2 = A_{3,n} + A_{4,n} + A_{5,n} + A_{6,n},$$

where

$$\begin{aligned}
A_{3,n} &\equiv \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} (V_{(i(\tau)-j)\Delta_n} - M_1(\zeta_{(i(\tau)-j)\Delta_n})) r_{i(\tau)-j}^2, \\
A_{4,n} &\equiv \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} (M_1(\zeta_{(i(\tau)-j)\Delta_n}) - M_1(\zeta_{(i(\tau)-j-1)\Delta_n})) r_{i(\tau)-j}^2, \\
A_{5,n} &\equiv \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} M_1(\zeta_{(i(\tau)-j-1)\Delta_n}) \left( r_{i(\tau)-j}^2 - \mathbb{E} \left[ r_{i(\tau)-j}^2 | \mathcal{F}_{i(\tau)-j-1} \right] \right), \\
A_{6,n} &\equiv \frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} M_1(\zeta_{(i(\tau)-j-1)\Delta_n}) \mathbb{E} \left[ r_{i(\tau)-j}^2 | \mathcal{F}_{i(\tau)-j-1} \right].
\end{aligned}$$

We now show that

$$A_{j,n} = o_p(1), \quad \text{for } j = 3, 4, 5. \quad (\text{A.13})$$

We note that  $A_{3,n}$  is an average of variables that are, conditionally on  $\mathcal{F}$ , independent with zero mean. Therefore,  $\mathbb{E} [A_{3,n}^2 | \mathcal{F}] \leq k_n^{-2} \Delta_n^{-2} \sum_{j=1}^{k_n} M_2(\zeta_{(i(\tau)-j)\Delta_n}) r_{i(\tau)-j}^4$ , which further implies  $\mathbb{E} [A_{3,n}^2] \leq K k_n^{-1} \rightarrow 0$ . Turning to  $A_{4,n}$ , we observe

$$\begin{aligned}
\mathbb{E} |A_{4,n}| &\leq \frac{K}{k_n \Delta_n} \sum_{j=1}^{k_n} \mathbb{E} \left[ \left| \zeta_{(i(\tau)-j)\Delta_n} - \zeta_{(i(\tau)-j-1)\Delta_n} \right| |r_{i(\tau)-j}|^2 \right] \\
&\leq \frac{K}{k_n \Delta_n} \sum_{j=1}^{k_n} \left( \mathbb{E} \left[ \left| \zeta_{(i(\tau)-j)\Delta_n} - \zeta_{(i(\tau)-j-1)\Delta_n} \right|^2 \right] \mathbb{E} \left[ r_{i(\tau)-j}^4 \right] \right)^{1/2} \\
&\leq K \Delta_n^{1/2} \rightarrow 0,
\end{aligned}$$

where the first inequality holds because  $M_1(\cdot)$  is locally Lipschitz and  $\zeta$  is bounded; the second inequality follows from the Cauchy–Schwarz inequality; the third inequality is due to the (1/2)-Hölder continuity of  $\zeta$  and  $\mathbb{E}[r_{i(\tau)-j}^4] \leq K \Delta_n^2$ . Finally, we note that  $A_{5,n}$  is an average of  $k_n$  martingale difference terms with bounded second moments. Therefore,  $\mathbb{E}[A_{5,n}^2] \leq K k_n^{-1} \rightarrow 0$ . The proof for (A.13) is now finished.

We now consider  $A_{6,n}$ . By Itô's formula and some standard estimates for continuous Itô semi-martingales (noting that  $r_{i(\tau)-j}$  does not contain jumps in restriction to  $\Omega_n$ ),

$$\begin{aligned}
&\mathbb{E} \left[ r_{i(\tau)-j}^2 | \mathcal{F}_{i(\tau)-j-1} \right] - \sigma_{(i(\tau)-j-1)\Delta_n}^2 \Delta_n \\
&= \int_{(i(\tau)-j-1)\Delta_n}^{(i(\tau)-j)\Delta_n} \mathbb{E} \left[ \sigma_s^2 - \sigma_{(i(\tau)-j-1)\Delta_n}^2 | \mathcal{F}_{(i(\tau)-j-1)\Delta_n} \right] ds + o_{pu}(\Delta_n),
\end{aligned}$$

where  $o_{pu}(\Delta_n)$  denotes a term that is  $o_p(\Delta_n)$  uniformly in  $j$ . Hence,

$$\begin{aligned} & \left| A_{6,n} - \frac{1}{k_n} \sum_{j=1}^{k_n} M_1(\zeta_{(i(\tau)-j-1)\Delta_n}) \sigma_{(i(\tau)-j-1)\Delta_n}^2 \right| \\ & \leq \frac{K}{k_n} \sum_{j=1}^{k_n} \frac{1}{\Delta_n} \int_{(i(\tau)-j-1)\Delta_n}^{(i(\tau)-j)\Delta_n} \mathbb{E} \left[ \left| \sigma_s^2 - \sigma_{(i(\tau)-j-1)\Delta_n}^2 \right| \middle| \mathcal{F}_{(i(\tau)-j-1)\Delta_n} \right] ds + o_p(1). \end{aligned}$$

We further note that the expectation of the first term on the majorant side of the above display is bounded by  $K \mathbb{E} \left[ \sup_{s \in [\tau-2k_n\Delta_n, \tau]} |\sigma_s^2 - \sigma_{\tau-}^2| \right]$ , which goes to zero since  $\sigma$  is càdlàg and bounded. Therefore,

$$A_{6,n} = \frac{1}{k_n} \sum_{j=1}^{k_n} M_1(\zeta_{(i(\tau)-j-1)\Delta_n}) \sigma_{(i(\tau)-j-1)\Delta_n}^2 + o_p(1) \xrightarrow{\mathbb{P}} M_1(\zeta_{\tau-}) \sigma_{\tau-}^2.$$

From here, it readily follows that

$$\Sigma_{n,\tau-}^{(12)} \xrightarrow{\mathbb{P}} 0.$$

We now turn to the limiting behavior of  $\Sigma_{n,\tau-}^{(22)}$ . For each  $i \in \{i(\tau) - j : 1 \leq j \leq k_n\}$ , we can decompose  $r_i = r'_i + r''_i$  in restriction to  $\Omega_n$  where

$$\begin{aligned} r'_i &= \sigma_{(i-1)\Delta_n} (W_{i\Delta_n} - W_{(i-1)\Delta_n}), \\ r''_i &= \int_{(i-1)\Delta_n}^{i\Delta_n} b_s ds + \int_{(i-1)\Delta_n}^{i\Delta_n} (\sigma_s - \sigma_{(i-1)\Delta_n}) dW_s. \end{aligned}$$

We can then decompose

$$\frac{1}{k_n \Delta_n^2} \sum_{j=1}^{k_n} r_{i(\tau)-j}^4 = A_{7,n} + A_{8,n}, \quad (\text{A.14})$$

where  $A_{7,n} \equiv \frac{1}{k_n \Delta_n^2} \sum_{j=1}^{k_n} r_{i(\tau)-j}'^4$  and  $A_{8,n}$  is defined implicitly by (A.14). Note that  $\mathbb{E}[r_i'^4 / \Delta_n^2 | \mathcal{F}_{(i-1)\Delta_n}] = 3\sigma_{(i-1)\Delta_n}^4$  and the variance of the martingale difference sequence  $r_i'^4 / \Delta_n^2 - \mathbb{E}[r_i'^4 / \Delta_n^2 | \mathcal{F}_{(i-1)\Delta_n}]$  is bounded. It is then easy to see that  $A_{7,n} \xrightarrow{\mathbb{P}} 3\sigma_{\tau-}^4$ . Turning to the term  $A_{8,n}$ , we first observe

$$|A_{8,n}| \leq \frac{K}{k_n \Delta_n^2} \sum_{j=1}^{k_n} \left| r_{i(\tau)-j}' \right|^3 \left| r_{i(\tau)-j}'' \right| + \frac{K}{k_n \Delta_n^2} \sum_{j=1}^{k_n} \left| r_{i(\tau)-j}'' \right|^4. \quad (\text{A.15})$$

By the Burkholder–Davis–Gundy inequality and Hölder’s inequality,

$$\mathbb{E} \left[ \left| r_{i(\tau)-j}'' \right|^4 \right] \leq K \Delta_n^4 + K \Delta_n \mathbb{E} \left[ \int_{(i(\tau)-j-1)\Delta_n}^{(i(\tau)-j)\Delta_n} (\sigma_s - \sigma_{(i(\tau)-j-1)\Delta_n})^4 ds \right]. \quad (\text{A.16})$$

Hence,

$$\begin{aligned} \mathbb{E} \left[ \frac{1}{k_n \Delta_n^2} \sum_{j=1}^{k_n} |r''_{i(\tau)-j}|^4 \right] &\leq K \Delta_n^2 + K \frac{1}{k_n \Delta_n} \mathbb{E} \left[ \int_{(i(\tau)-k_n-1)\Delta_n}^{(i(\tau)-1)\Delta_n} (\sigma_s - \sigma_{(i(\tau)-j-1)\Delta_n})^4 ds \right] \\ &\leq K \Delta_n^2 + K \mathbb{E} \left[ \sup_{t \in [\tau-2k_n\Delta_n, \tau)} |\sigma_t - \sigma_{\tau-}|^4 \right] \rightarrow 0, \end{aligned}$$

where the convergence follows from the bounded convergence theorem, because  $\sigma$  is càdlàg and bounded. Therefore, the second term on the majorant side of (A.15) is  $o_p(1)$ . Since  $A_{\tau,n} = O_p(1)$ , we can use Hölder's inequality to show that the first term on majorant side of (A.15) is also  $o_p(1)$ . Hence,  $A_{8,n} = o_p(1)$  and  $k_n^{-1} \Delta_n^{-2} \sum_{j=1}^{k_n} r_{i(\tau)-j}^4 \xrightarrow{\mathbb{P}} 3\sigma_{\tau-}^4$ . Recall that  $\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{i(\tau)-j}^2 \xrightarrow{\mathbb{P}} \sigma_{\tau-}^2$ . Hence,  $\Sigma_{n,\tau-}^{(22)} \xrightarrow{\mathbb{P}} 2\sigma_{\tau-}^4$  as asserted. The proof for (A.12) is now complete.

Step 3. We finish the proof of part (b) of Theorem 1 in this step. By essentially the same argument for (A.10), we can show that

$$\sqrt{k_n}(\hat{m}_{n,\tau}^* - \hat{m}_{n,\tau}, \hat{\sigma}_{n,\tau}^* - \hat{\sigma}_{n,\tau}) \xrightarrow{\mathcal{L}|\mathcal{G}} (\eta_\tau, \eta'_\tau). \quad (\text{A.17})$$

Moreover, since the variables in (A.10) and (A.17) are  $\mathcal{G}$ -conditionally independent between the pre- and post-event windows and across  $\tau \in \mathcal{T}$  (by the construction of the local i.i.d. resampling scheme), the joint convergence holds as well. Therefore,

$$\sqrt{k_n}(\tilde{\mathbf{S}}_n^* - \tilde{\mathbf{S}}_n) \xrightarrow{\mathcal{L}|\mathcal{G}} (\eta_{\tau-}, \eta_\tau, \eta'_{\tau-}, \eta'_\tau, 0)_{\tau \in \mathcal{T}}. \quad (\text{A.18})$$

Note that, with probability approaching one,  $\tilde{\mathbf{S}}_n$  falls in the neighborhood of  $\tilde{\mathbf{S}}$  on which  $f$  is continuously differentiable. The assertion of the theorem then follows from the delta method. *Q.E.D.*

## Appendix B: Additional data description

### B.1 Macroeconomic news announcements

Table B.1: Other macroeconomic announcements

Category by Bloomberg	Index	No.Obs.	Time <sup>†</sup>
Labor Market	JOLTS Job Openings	47	10:00
Retail and Wholesale	Wards Total Vehicle Sales	10	14:41
Retail and Wholesale	Wholesale Inventories	160	10:00
Cyclical Indicators	Chicago Purchasing Manager	69	10:00
Cyclical Indicators	Dallas Fed Manf Activity	68	10:30
Cyclical Indicators	ISM Milwaukee	58	10:00
Cyclical Indicators	Kansas City Fed Manf Activity	37	11:00
Cyclical Indicators	Leading Index	161	10:00
Cyclical Indicators	Philadelphia Fed Business Outlook	162	10:00
Cyclical Indicators	Richmond Fed Manufact Index	108	10:00
Cyclical Indicators	U of Mich Sentiment	78	10:00
Personal Household Sector	Consumer Credit	159	15:00
Housing and Real Estate	Existing Home Sales	116	10:00
Housing and Real Estate	FHFA House Price Index	56	10:00
Housing and Real Estate	House Price Purchase Index	16	10:00
Housing and Real Estate	NAHB Housing Market Index	138	13:00
Housing and Real Estate	New Home Sales	160	10:00
Housing and Real Estate	One Family Home Resales	47	10:00
Housing and Real Estate	Pending Home Sales	114	10:00
Government Finance And Debt	Monthly Budget Statement	155	14:00

*Notes:* The table lists all of the intraday news announcements included in the “Others” category. †Some of these release times vary over the sample. To ensure that the pre- and post-event windows both have 30 observations, we only keep announcements between 10:00 and 15:30. The times indicated in the table refer to the most common release times.

### B.2 A detailed description for the FOMC sentiment measure

Our detailed analysis of the volume-volatility relationship around FOMC announcements in Section 6.4 uses a textual measure of the negative sentiment in the actual written statements accompanying

Table B.2: Word counts in FOMC Sentiment

Word	unemployment	slow	weak	decline	against	depressed	downward	diminish	concern	persist
Count	73	65	53	39	23	23	21	18	16	16

*Notes:* The table reports the counts of the ten most frequently occurring financial-negative words in the FOMC statements over the April, 2001 to September, 2014 sample period.

each of the FOMC rate decisions. Our construction of this sentiment measure closely follows Loughran and McDonald (2011), henceforth LM.

Specifically, we begin by extracting the actual text for each of the FOMC statements in our sample, excluding the paragraph detailing the voting decisions and the paragraph stating the new target funds rate. The individual words in each of the statements are then compared with the Fin-Neg list of LM, resulting in a total of 79 different negative words. Table B.2 provides a count of ten such words that occur most frequently over the full sample. The count of words that are of different forms of the same word (i.e., inflections) are summed up. For example, the 65 count for the word “slow” also includes occurrences of “slow,” “slowed,” “slower,” “slowing,” and “slowly.”

Following LM, we then construct a weighted measure for the  $i$ th negative word in the  $j$ th statement according to the following formula:

$$w_{i,j} \equiv \begin{cases} \frac{1 + \log(tf_{i,j})}{1 + \log(a_j)} \log\left(\frac{N}{df_i}\right), & tf_{i,j} \geq 1, \\ 0, & tf_{i,j} \leq 1, \end{cases} \quad (\text{B.1})$$

where  $tf_{i,j}$  refers to the raw count of the  $i$ th word in the  $j$ th statement,  $a_j$  denotes the total number of words in the  $j$ th statement, and  $df_i$  is the number of statements out of the total number of statements  $N$  containing at least one occurrence of the  $i$ th word. Importantly, this non-linear weighting scheme implies that negative words that seldom occur will receive a higher weight than more commonly used negative words. The overall negative sentiment for the  $j$ th statement is then simply obtained by summing these individual weights over all of the 79 negative words that occurred over the full sample, that is,

$$\text{FOMC Sentiment}_j = \sum_{i=1}^{79} w_{i,j}. \quad (\text{B.2})$$

The words included in the Fin-Neg list are, of course, somewhat subjective. Also, certain words do not necessarily have a negative meaning in the context of FOMC announcements. For example, the most frequently used word “unemployment,” which may sound negative in the 10-K filings analyzed by LM, may be a neutral word in the FOMC statements, simply used to summarize the economic conditions, whether good or bad. Similarly, the word “late” does not have a negative



meaning when used in a sentence like: "... are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014." To guard against such ambiguities, we explicitly checked the usage of each of the words on the Fin-Neg list in the FOMC statements, and then select only those occurrences whose meaning were unambiguously negative. The resulting "selective" measure based on a total of 67 different words is highly correlated with FOMC Sentiment (correlation coefficient = 0.73). All of the results based on this more selective sentiment measure are very close to those based on the original FOMC Sentiment measure reported in Section 6.4 of the main text. These additional results are available upon request.

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**Supplemental Appendix to:  
Volume, Volatility and Public News Announcements**

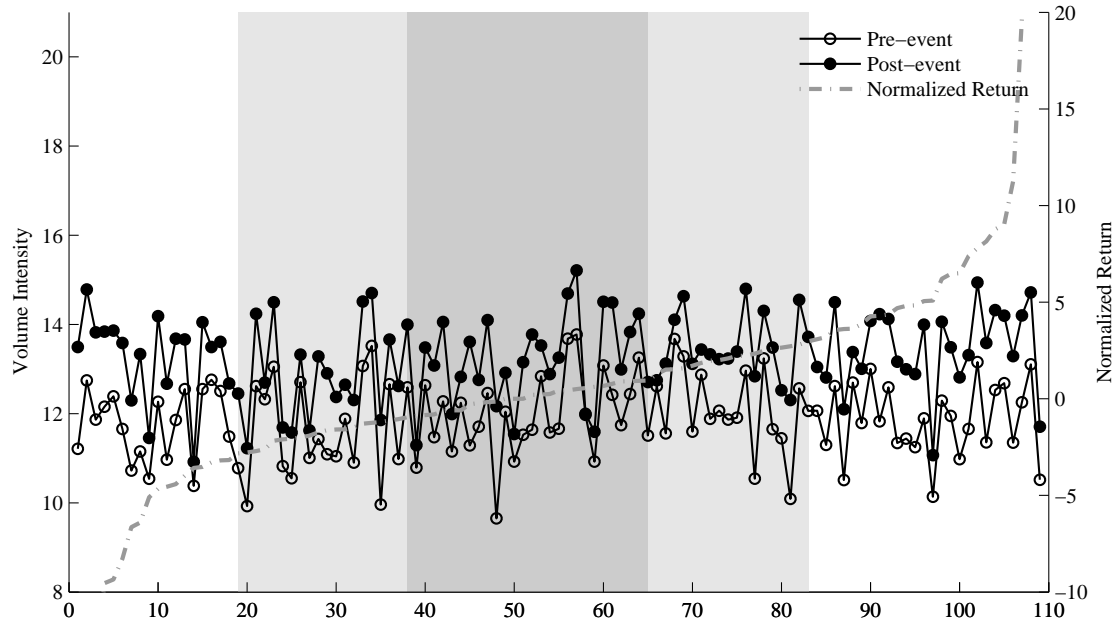
This version: June 22, 2016

Tim Bollerslev, Jia Li and Yuan Xue

## S.1 Volume intensity sorted by five-minute normalized returns

Figure S.1, below, shows that the patterns seen in Figure 5 in the main text is robust to changing the sampling frequency from one minute to five minutes.

Figure S.1: Sorted volume around FOMC announcements



*Note:* The figure shows the pre- and post-event log volume intensities sorted on the basis of the 5-minute normalized returns  $r_{i(\tau)}/\hat{\sigma}_{\tau-\sqrt{\Delta_n}}$ ,  $\Delta_n = 5$  minutes, around FOMC announcements (dots). The normalized return (dash-dotted line) increases from left to right. Announcements with normalized returns less than 1 (resp. 3) are highlighted by the dark (resp. light) shaded area.

## S.2 Raw estimates without using the DID method

Tables S.1, S.2 and S.3, below, are the “no-DID” counterparts of Tables 4, 5 and 6 in the main text, respectively. Here, we report the “raw” estimates  $\hat{\theta}$  (see (5.2)), instead of  $\tilde{\theta}$  obtained using the DID method. We note that the raw estimates of the intercept terms in Tables S.1 and S.2 often become (counterintuitively) negative, which re-affirms the pattern reported in Table 3 of the main text, and further highlights the importance of correcting the intraday U-shape pattern using the DID approach as we did in the main text.

Table S.1: Volume-volatility elasticities around public news announcements (no DID)

	FOMC	ISMM	ISMNM	CC	Others
Constant ( $a_0$ )	0.689** (0.077)	-0.043** (0.018)	-0.096** (0.016)	-0.029 (0.018)	-0.065** (0.005)
Elasticity ( $a_1$ )	0.662** (0.063)	0.614** (0.062)	0.704** (0.069)	0.681** (0.074)	0.857** (0.022)
$R^2$	0.317	0.216	0.256	0.185	0.313

*Notes:* The table reports the results from the “no-DID” regression in equation (5.2) for the specification  $\Delta \widehat{\log(m_\tau)} = a_0 - a_1 \cdot \Delta \widehat{\log(\sigma_\tau)}$ . The sample spans the period from April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \*\* indicates significance at the 1% level.

Table S.2: Volume-volatility elasticity estimates and disagreement measures (no DID)

<i>Baseline estimates:</i>					
Constant ( $a_0$ )	-0.067**	-0.068**	-0.069**	-0.070**	-0.070**
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Elasticity ( $a_1$ )	0.800**	0.858**	0.964**	0.952**	1.010**
	(0.018)	(0.022)	(0.041)	(0.034)	(0.044)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>					
<i>News-category dummy variables:</i>					
FOMC		-0.196**	-0.199**	-0.191**	-0.194**
		(0.065)	(0.065)	(0.066)	(0.065)
ISMM		-0.238**	-0.237**	-0.233**	-0.233**
		(0.067)	(0.067)	(0.068)	(0.068)
ISMNM		-0.153*	-0.153*	-0.149*	-0.150*
		(0.073)	(0.073)	(0.073)	(0.073)
CC		-0.171*	-0.177*	-0.160*	-0.166*
		(0.073)	(0.073)	(0.073)	(0.074)
<i>Disagreement measures:</i>					
Dispersion			-0.041**		-0.030*
			(0.013)		(0.014)
Weekly Policy				-0.052**	-0.042**
				(0.013)	(0.014)
$R^2$	0.539	0.54	0.541	0.541	0.542

*Notes:* The table reports the results from the “no-DID” regression in (5.2) for the specification  $\Delta \widehat{\log}(m_\tau) = a_0 + b_0^\top X_{0,\tau} + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \widehat{\log}(\sigma_\tau)$  based on all of the public announcements. In all specifications,  $X_{0,\tau}$  include category dummy variables for FOMC rate decision (FOMC), ISM Manufacturing Index (ISMM), ISM Non-Manufacturing Index (ISMNM) and Consumer Confidence Index (CC); the estimates of these dummies (i.e.,  $b_0$ ) are not reported for brevity. The Dispersion variable is constructed as the latest forecast dispersion of the one-quarter-ahead unemployment rate from the Survey of Professional Forecasters before the announcement. The Weekly Policy variable is constructed as the weekly moving average of the economic policy uncertainty index developed by Baker et al. (2015) before the announcement. Both variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.



Table S.3: Volume-volatility elasticity estimates around FOMC announcements (no DID)

<i>Baseline estimates:</i>						
Constant ( $a_0$ )	0.689**	0.692**	0.690**	0.641**	0.653**	0.642**
	(0.076)	(0.075)	(0.076)	(0.075)	(0.075)	(0.076)
Elasticity ( $a_1$ )	0.662**	0.897**	0.712**	0.865**	1.011**	0.976**
	(0.063)	(0.077)	(0.074)	(0.077)	(0.086)	(0.086)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>						
Dispersion		-0.095**			-0.075**	-0.085**
		(0.018)			(0.018)	(0.019)
Weekly Policy			-0.027			0.055*
			(0.019)			(0.024)
FOMC Sentiment				-0.105**	-0.084**	-0.106**
				(0.022)	(0.022)	(0.025)
$R^2$	0.317	0.362	0.314	0.363	0.388	0.392

*Notes:* The table reports the results from the “no-DID” regression in equation (5.2) for the specification  $\Delta \widehat{\log(m_\tau)} = a_0 + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \widehat{\log(\sigma_\tau)}$  based on FOMC announcements. Dispersion and Weekly Policy are constructed as in Table 5. FOMC Sentiment is a textual measure constructed using financial-negative words in the FOMC press release. These variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

### S.3 Results for DIA ETF

Tables S.4 and S.5, below, are counterparts of Tables 5 and 6 in the main text, but with the SPY ETF replaced by the DIA ETF, which tracks the Dow Jones Industrial Average index. Our main findings using the SPY are re-affirmed using the DIA data, which we summarize as follows. Firstly, the volume-volatility elasticity is significantly below unity when we do not control for the level of disagreement. Secondly, the proxies of disagreement impact negatively the volume-volatility elasticity, although the Dispersion variable does not have a significant effect for the DIA data. Thirdly, after controlling for disagreement in the elasticity specification, the baseline elasticity is not statistically different from the no-disagreement benchmark of unity. Finally, for FOMC announcements, the FOMC sentiment measure negatively impacts the volume-volatility elasticity, and shows better performance than the “catch-all” Weekly Policy measure.

Table S.4: Volume-volatility elasticity estimates and disagreement measures (DIA)

<i>Baseline estimates:</i>					
Constant ( $a_0$ )	0.056** (0.010)	0.057** (0.010)	0.057** (0.010)	0.057** (0.010)	0.057** (0.010)
Elasticity ( $a_1$ )	0.786** (0.026)	0.775** (0.031)	0.853** (0.062)	0.948** (0.052)	0.956** (0.068)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>					
<i>News-category dummy variables:</i>					
FOMC		0.086 (0.096)	0.087 (0.097)	0.091 (0.097)	0.091 (0.097)
ISMM		0.265** (0.101)	0.262** (0.101)	0.281** (0.100)	0.281** (0.100)
ISMNM		-0.128 (0.102)	-0.128 (0.102)	-0.121 (0.102)	-0.121 (0.102)
CC		-0.114 (0.106)	-0.109 (0.106)	-0.087 (0.107)	-0.087 (0.107)
<i>Disagreement measures:</i>					
Dispersion			-0.030 (0.019)		-0.004 (0.020)
Weekly Policy				-0.096** (0.019)	-0.095** (0.020)
$R^2$	0.364	0.365	0.365	0.368	0.367

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\widehat{\Delta \log(m_\tau)} = a_0 + b_0^\top X_{0,\tau} + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on all of the public announcements, using the past 22 non-announcement days as the control group. In all specifications,  $X_{0,\tau}$  include category dummy variables for FOMC rate decision (FOMC), ISM Manufacturing Index (ISMM), ISM Non-Manufacturing Index (ISMNM) and Consumer Confidence Index (CC); the estimates of  $b_0$  are not reported for brevity. The Dispersion variable is constructed as the latest forecast dispersion of the one-quarter-ahead unemployment rate from the Survey of Professional Forecasters before the announcement. The Weekly Policy variable is constructed as the weekly moving average of the economic policy uncertainty index developed by Baker et al. (2015) before the announcement. Both variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

Table S.5: Volume-volatility elasticity estimates around FOMC announcements (DIA)

<i>Baseline estimates:</i>						
Constant ( $a_0$ )	0.621**	0.617**	0.623**	0.597**	0.598**	0.592**
	(0.098)	(0.098)	(0.099)	(0.099)	(0.099)	(0.099)
Elasticity ( $a_1$ )	0.861**	1.004**	0.903**	1.013**	1.081**	1.060**
	(0.089)	(0.119)	(0.108)	(0.106)	(0.126)	(0.123)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>						
Dispersion		-0.055			-0.034	-0.039
		(0.030)			(0.030)	(0.032)
Weekly Policy			-0.023			0.029
			(0.032)			(0.042)
FOMC Sentiment				-0.085**	-0.074*	-0.086*
				(0.032)	(0.032)	(0.038)
$R^2$	0.298	0.300	0.293	0.310	0.306	0.301

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(\widetilde{m}_\tau) = a_0 + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\widetilde{\sigma}_\tau)$  based on FOMC announcements, using the past 22 non-announcement days as the control group. Dispersion and Weekly Policy are constructed as in Table 5. FOMC Sentiment is a textual measure constructed using financial-negative words in the FOMC press release. These variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

## S.4 Alternative dispersion and uncertainty measures

In this section, we provide robustness checks by replacing the disagreement proxies with similar alternatives as follows: (i) we use the one-year-head unemployment and GDP growth rates in place of the one-quarter-ahead unemployment rate when constructing the Dispersion measure; (ii) we use the Monthly Policy measure in place of the Weekly Policy measure. As the tables below show, our findings in the main text are robustness to these changes.

Table S.6: Volume-volatility elasticity estimates and disagreement measures based on one-year-ahead unemployment dispersion

<i>Baseline estimates:</i>					
Constant ( $a_0$ )	0.044**	0.041**	0.041**	0.041**	0.041**
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
Elasticity ( $a_1$ )	0.733**	0.776**	0.921**	0.921**	0.990**
	(0.020)	(0.024)	(0.043)	(0.035)	(0.046)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>					
<i>News-category dummy variables:</i>					
FOMC		-0.060	-0.065	-0.048	-0.053
		(0.073)	(0.072)	(0.073)	(0.073)
ISMM		-0.238**	-0.244**	-0.220**	-0.226**
		(0.067)	(0.067)	(0.067)	(0.067)
ISMNM		-0.090	-0.086	-0.082	-0.081
		(0.073)	(0.073)	(0.072)	(0.073)
CC		-0.244**	-0.248**	-0.202**	-0.210**
		(0.077)	(0.078)	(0.078)	(0.078)
<i>Disagreement measures:</i>					
Dispersion			-0.055**		-0.033*
			(0.013)		(0.014)
Weekly Policy				-0.079**	-0.069**
				(0.013)	(0.014)
$R^2$	0.481	0.482	0.484	0.486	0.486

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + b_0^\top X_{0,\tau} + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on all of the public announcements, using the past 22 non-announcement days as the control group. In all specifications,  $X_{0,\tau}$  include category dummy variables for FOMC rate decision (FOMC), ISM Manufacturing Index (ISMM), ISM Non-Manufacturing Index (ISMNM) and Consumer Confidence Index (CC); the estimates of  $b_0$  are not reported for brevity. The Dispersion variable is constructed as the latest forecast dispersion of the one-year-ahead unemployment rate from the Survey of Professional Forecasters before the announcement. The Weekly Policy variable is constructed as the weekly moving average of the economic policy uncertainty index developed by Baker et al. (2015) before the announcement. Both variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

Table S.7: Volume-volatility elasticity estimates around FOMC announcements based on one-year-ahead unemployment dispersion

<i>Baseline estimates:</i>						
Constant ( $a_0$ )	0.586**	0.592**	0.583**	0.545**	0.557**	0.555**
	(0.076)	(0.076)	(0.076)	(0.075)	(0.076)	(0.076)
Elasticity ( $a_1$ )	0.716**	0.911**	0.790**	0.913**	0.986**	0.960**
	(0.065)	(0.083)	(0.081)	(0.081)	(0.088)	(0.089)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>						
Dispersion		-0.077**			-0.044*	-0.049*
		(0.019)			(0.021)	(0.022)
Weekly Policy			-0.037			0.032
			(0.021)			(0.028)
FOMC Sentiment				-0.104**	-0.084**	-0.096**
				(0.022)	(0.024)	(0.027)
$R^2$	0.330	0.354	0.329	0.370	0.373	0.37

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on FOMC announcements, using the past 22 non-announcement days as the control group. Dispersion and Weekly Policy are constructed as in Table S.6. FOMC Sentiment is a textual measure constructed using financial-negative words in the FOMC press release. These variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

Table S.8: Volume-volatility elasticity estimates and disagreement measures based on one-year-ahead GDP dispersion

<i>Baseline estimates:</i>					
Constant ( $a_0$ )	0.044**	0.041**	0.040**	0.041**	0.041**
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
Elasticity ( $a_1$ )	0.733**	0.776**	0.938**	0.921**	0.973**
	(0.020)	(0.023)	(0.061)	(0.037)	(0.061)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>					
<i>News-category dummy variables:</i>					
FOMC		-0.060	-0.056	-0.048	-0.048
		(0.069)	(0.069)	(0.069)	(0.069)
ISM		-0.238**	-0.238**	-0.220**	-0.221**
		(0.071)	(0.071)	(0.071)	(0.071)
ISMNM		-0.090	-0.083	-0.082	-0.080
		(0.075)	(0.075)	(0.075)	(0.075)
CC		-0.244**	-0.246**	-0.202**	-0.206**
		(0.076)	(0.076)	(0.076)	(0.076)
<i>Disagreement measures:</i>					
Dispersion			-0.040**		-0.015
			(0.014)		(0.015)
Weekly Policy				-0.079**	-0.074**
				(0.014)	(0.015)
$R^2$	0.481	0.482	0.483	0.486	0.485

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + b_0^\top X_{0,\tau} + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on all of the public announcements, using the past 22 non-announcement days as the control group. In all specifications,  $X_{0,\tau}$  include category dummy variables for FOMC rate decision (FOMC), ISM Manufacturing Index (ISM), ISM Non-Manufacturing Index (ISMNM) and Consumer Confidence Index (CC); the estimates of  $b_0$  are not reported for brevity. The Dispersion variable is constructed as the latest forecast dispersion of the one-year-ahead GDP growth rate from the Survey of Professional Forecasters before the announcement. The Weekly Policy variable is constructed as the weekly moving average of the economic policy uncertainty index developed by Baker et al. (2015) before the announcement. Both variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.



Table S.9: Volume-volatility elasticity estimates around FOMC announcements based on one-year-ahead GDP dispersion

<i>Baseline estimates:</i>						
Constant ( $a_0$ )	0.586**	0.582**	0.583**	0.545**	0.546**	0.539**
	(0.078)	(0.077)	(0.078)	(0.078)	(0.078)	(0.078)
Elasticity ( $a_1$ )	0.716**	1.016**	0.790**	0.913**	1.148**	1.154**
	(0.067)	(0.102)	(0.080)	(0.083)	(0.108)	(0.108)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>						
Dispersion		-0.073**			-0.061**	-0.078**
		(0.021)			(0.021)	(0.024)
Weekly Policy			-0.037			0.058
			(0.020)			(0.031)
FOMC Sentiment				-0.104**	-0.096**	-0.121**
				(0.022)	(0.023)	(0.027)
$R^2$	0.33	0.349	0.329	0.37	0.382	0.385

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on FOMC announcements, using the past 22 non-announcement days as the control group. Dispersion and Weekly Policy are constructed as in Table S.8. FOMC Sentiment is a textual measure constructed using financial-negative words in the FOMC press release. These variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

Table S.10: Volume-volatility elasticity estimates and disagreement measures based on Monthly Policy

<i>Baseline estimates:</i>					
Constant ( $a_0$ )	0.044**	0.041**	0.040**	0.041**	0.041**
	(0.007)	(0.006)	(0.006)	(0.007)	(0.007)
Elasticity ( $a_1$ )	0.733**	0.776**	0.906**	0.945**	1.001**
	(0.020)	(0.024)	(0.043)	(0.038)	(0.046)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>					
<i>News-category dummy variables:</i>					
FOMC		-0.060	-0.058	-0.056	-0.055
		(0.072)	(0.072)	(0.072)	(0.072)
ISMM		-0.238**	-0.238**	-0.217**	-0.219**
		(0.070)	(0.070)	(0.070)	(0.070)
ISMNM		-0.090	-0.092	-0.086	-0.088
		(0.076)	(0.076)	(0.077)	(0.077)
CC		-0.244**	-0.244**	-0.227**	-0.229**
		(0.072)	(0.072)	(0.072)	(0.072)
<i>Disagreement measures:</i>					
Dispersion			-0.051**		-0.029*
			(0.013)		(0.014)
Monthly Policy				-0.083**	-0.074**
				(0.013)	(0.014)
$R^2$	0.481	0.482	0.483	0.486	0.486

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + b_0^\top X_{0,\tau} + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on all of the public announcements, using the past 22 non-announcement days as the control group. In all specifications,  $X_{0,\tau}$  include category dummy variables for FOMC rate decision (FOMC), ISM Manufacturing Index (ISMM), ISM Non-Manufacturing Index (ISMNM) and Consumer Confidence Index (CC); the estimates of  $b_0$  are not reported for brevity. The Dispersion variable is constructed as the latest forecast dispersion of the one-quarter-ahead unemployment rate from the Survey of Professional Forecasters before the announcement. The Monthly Policy variable is constructed as the monthly moving average of the economic policy uncertainty index developed by Baker et al. (2015) before the announcement. Both variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.

Table S.11: Volume-volatility elasticity estimates around FOMC announcements based on Monthly Policy

<i>Baseline estimates:</i>						
Constant ( $a_0$ )	0.586**	0.579**	0.586**	0.545**	0.550**	0.539**
	(0.080)	(0.078)	(0.079)	(0.078)	(0.078)	(0.079)
Elasticity ( $a_1$ )	0.716**	0.996**	0.799**	0.913**	1.089**	1.059**
	(0.068)	(0.087)	(0.076)	(0.080)	(0.092)	(0.090)
<i>Estimates for explanatory variables in elasticity (<math>b_1</math>):</i>						
Dispersion		-0.108**			-0.088**	-0.099**
		(0.021)			(0.021)	(0.023)
Monthly Policy			-0.040*			0.052
			(0.019)			(0.027)
FOMC Sentiment				-0.104**	-0.077**	-0.103**
				(0.022)	(0.022)	(0.027)
$R^2$	0.33	0.382	0.331	0.37	0.4	0.402

*Notes:* The table reports the results from the DID regression in equation (5.4) for the specification  $\Delta \log(m_\tau) = a_0 + (a_1 + b_1^\top X_{1,\tau}) \cdot \Delta \log(\sigma_\tau)$  based on FOMC announcements, using the past 22 non-announcement days as the control group. Dispersion and Monthly Policy are constructed as in Table S.10. FOMC Sentiment is a textual measure constructed using financial-negative words in the FOMC press release. These variables are scaled by their own sample standard deviations. The sample spans April 10, 2001 to September 30, 2014. Bootstrapped standard errors are reported in parentheses. \* and \*\* indicate significance at the 5% and 1% level, respectively.