Default Cycles*

Wei Cui† Leo Kaas‡

December 2016

Abstract

Corporate default rates are counter-cyclical and are often clustered over prolonged episodes. This paper develops a tractable macroeconomic model in which persistent default cycles are the outcome of variations in self-fulfilling beliefs about credit market conditions. Interest spreads and leverage ratios are determined in optimal debt contracts that reflect the expected default risk of borrowing firms. Next to sunspot shocks, the model also features other financial shocks that are unrelated to default risk. We calibrate the model to evaluate the impact of the different financial shocks on the credit market and on output dynamics. Self-fulfilling credit market expectations trigger sizeable reactions in default rates and generate endogenously persistent credit and output cycles. All credit market shocks together account for over 80% of the variance of U.S. GDP growth during 1982–2015.

JEL classification: E22, E32, E44, G12

Keywords: Firm default; Financing constraints; Credit Spreads; Sunspots

Preliminary and Incomplete Draft

*We thank Lars Hansen, Franck Portier and Morten Ravn for helpful comments and discussions, and the audience at the UCL macroeconomics workshop (November 2016) for their comments. Leo Kaas thanks the German Research Foundation (grant No. KA 442/15) for financial support. The usual disclaimer applies.

†Department of Economics, University College London, 30 Gordon Street, London, WC1H0AX, UK, and Centre for Macroeconomics, UK. Email: w.cui@ucl.ac.uk

‡Department of Economics, University of Konstanz, 78457 Konstanz, Germany. Email: leo.kaas@uni-konstanz.de

1 Introduction

Many recessions are accompanied by substantial increases of corporate default rates and credit spreads, together with declines of business credit. On the one hand, corporate defaults tend to be clustered over prolonged episodes which gives rise to persistent default cycles (see e.g. Giesecke, Longstaff, Schaefer, and Strebulaev (2011)). Such clustering of default can only partly be explained by observable firm-specific or macroeconomic variables, but is driven by unobserved factors that are correlated across firms and over time (Duffie, Eckner, Horel, and Saita (2009)). On the other hand, credit spreads tend to lead the cycle and are not fully accounted for by expected default. Moreover, less than half of the volatility of credit spreads can be explained by expected default losses; instead, it is the "excess premium" on corporate bonds that has the strongest impact on investment and output (Gilchrist and Zakrajšek (2012)).

This paper examines the joint dynamics of firm default, credit spreads and output, using a tractable dynamic general equilibrium model in which firms issue defaultable debt. We argue that default rates in such economies are susceptible to self-fulfilling beliefs over credit conditions. States of low default and good credit conditions can alternate between states of high default and bad credit conditions. Low-frequency variation of self-fulfilling beliefs play a key role in accounting for the persistent dynamics of default rates and their co-movement with macroeconomic variables. However, we find that these self-fulfilling beliefs cannot account for the volatility of credit spreads which must be driven by other financial shocks, such as disturbances to the cost of financial intermediation.

To illustrate our main idea, we present in Section 3 a simple partial-equilibrium model of firm credit with limited commitment and equilibrium default. Leverage and the interest rate spread depend on the value that borrowing firms attach to future credit market conditions which critically impacts the firms' default decisions, and hence is taken into account in the optimal credit contract. This credit market value is a forward-looking variable which reacts to self-fulfilling expectations. A well-functioning credit market with a low interest rate and a low default rate is highly valuable for firms, and this high valuation makes credit contracts with few defaults self-enforcing. Conversely, a weak credit market with a higher interest rate and more default is valued less by firms, and therefore it cannot sustain credit contracts that prevent high default rates.

After this illustrative example, we build in Section 4 a general-equilibrium model in order to analyze the role of self-fulfilling expectations and fundamental shocks for the dynamics of default rates, spreads and their relationships with the macro economy. Credit constraints, spreads, default rates, and aggregate productivity are all endogenous outcomes of optimal debt contracts. As in the simple model, leverage ratios and default rates depend on the value that borrowers attach to future credit market conditions which is a forward-looking variable. Aggregate productivity is determined by the reallocation of existing capital among heterogenous firms which depends on current leverage ratios and on past default events. When leverages are tightened or when more firms opt for default, capital reallocation slows down and aggregate productivity falls.

The model features heterogeneous firms which differ in productivity and in their access to the credit market. High-productivity firms with a good credit standing borrow up to an endogenous credit limit at an interest rate which partly reflects the expected default loss and which also includes an excess interest premium. This premium, which may itself be subject to aggregate shocks, is a shortcut to account for the so-called "credit spread puzzle" according to which actual credit spreads are larger than expected default losses (e.g., Elton, Gruber, Agrawal, and Mann (2001) and Huang and Huang (2012)).[1] We also allow the recovery rate to fluctuate which affects the expected default loss and hence takes a direct impact on leverage and on the predicted component of the credit spread.

If a firm opts for default, a fraction of its assets can be recovered by creditors. After default, the firm's owner may continue to operate a business (possibly under a different name), but he/she loses the good credit standing and hence remains temporarily excluded from the credit market. Because the number of firms with credit market access is endogenous and aggregate factor productivity depends on the capital allocation among heterogeneous firms, periods of high default can have a long-lasting impact on credit, aggregate productivity, and output. Modeling the default cycle is therefore crucial for the overall macroeconomic dynamics.

In Section 5 we calibrate this model and show that it exhibits a natural equilibrium indeterminacy which gives rise to endogenous cycles driven by self-fulfilling beliefs in credit market conditions (sunspot shocks). In a quantitative analysis we examine how the model economy responds to such shocks, as well as to fundamental shocks to the financial sector (shocks to the recovery rate or to the excess premium). We show that sunspot shocks are crucial for the dynamics of default rates. All three financial shocks together explain the GDP cycle since 1982 rather well and account for 83% of output-growth volatility.

Our work relates to a number of recent contributions analyzing the macroeconomic implications of credit spreads and firm default. Building on Bernanke, Gertler, and Gilchrist (1999), Christiano, Motto, and Rostagno (2014) introduce risk shocks in a quantitative

[1]For instance, the excess premium can reflect intermediation costs, liquidity, tax or risk premia (cf. Gilchrist and Zakrajšek (2012)). We do not model the underlying sources of volatility for this variable.

business-cycle model and show that these shocks not only generate counter-cyclical spreads but also account for a large fraction of macroeconomic fluctuations. Miao and Wang (2010) include long-term defaultable debt in a macroeconomic model with financial shocks to the recovery rate. In line with empirical evidence, they find that credit spreads are counter-cyclical and lead output and stock returns. Gomes and Schmid (2012) develop a macroeconomic model with endogenous default of heterogeneous firms and analyze the dynamics of credit spreads. Gourio (2013) is motivated by the volatility of the excess bond premium and argues that time-varying risk of rare depressions (disaster risk) can generate plausible volatility of credit spreads and co-movement with macroeconomic variables. Self-fulfilling expectations do not matter in all these contributions which differ from our model in that default incentives do not depend on expected credit conditions. Also, these papers do not allow for a link between the credit market and aggregate factor productivity.[2]

Our work further builds on a literature on self-fulfilling expectations and multiplicity in macroeconomic models with financial market imperfections. Most closely related is Azariadis, Kaas, and Wen (2016) who show that sunspot shocks account for the pro-cyclical dynamics of unsecured credit. As in this paper, equilibrium indeterminacy arises due to a dynamic complementarity in borrowers' valuation of credit market access, but in their model there is no default in equilibrium and credit spreads are zero. Harrison and Weder (2013), Benhabib and Wang (2013), Liu and Wang (2014) and Gu, Mattesini, Monnet, and Wright (2013) also show how equilibrium indeterminacy and endogenous credit cycles arise in credit-constrained economies. None of these papers addresses default and credit spreads.

The co-existence of equilibria with high (low) interest rates and high (low) default rates relates to a literature on self-fulfilling sovereign debt crises. In a two-period model, Calvo (1988) shows how multiple equilibria emerge from a positive feedback between interest rates and debt levels. Lorenzoni and Werning (2013) extend this idea to a dynamic setting to study the role of fiscal policy rules and debt accumulation for the occurrence of debt crises. On the other hand, Cole and Kehoe (2000) find that self-fulfilling debt crises occur because governments cannot roll over their debt (cf. Conesa and Kehoe (2015) Aguiar, Amador, Farhi, and Gopinath (2013)). Our mechanism for multiplicity is different from these contributions by emphasizing the role of expectations about future credit conditions. The credit conditions determine default incentives, interest rates, and leverage ratios in an optimal contracting framework, although we restrict ourselves to standard debt contracts as they are common in reality.

[2]Khan, Senga, and Thomas (2016) introduce firm dynamics and default risk in a macroeconomic model and show that counter-cyclical default affect the capital allocation among firms, which amplifies and propagate real and financial shocks. Unlike our model, there is no role for self-fulfilling expectations.

2 VAR Evidence

We obtain data for the recovery rate and the all-rated default rate for Moody-rated corporate bonds, covering the period 1982 to 2015, all in percentage terms. The data set is contained in the 2015 annual report published by Moody's Investors Service. The recovery rate is measured by the post-default bond price. For the interest spread, we use the credit spread index developed by Gilchrist and Zakrajšek (2012) that is representative for the full corporate bond market.[3] Finally, aggregate real GDP (in 2009 dollars) is measured by the sum of private consumption and private non-residential investment in the national accounts. We deflate by the GDP deflator to measure GDP in equivalent 2009 dollars.

Table 1 shows the correlation structure of these three variables. As expected, default rate and spreads are highly positively correlated, and both of them are countercyclical. The recovery rate is highly negatively correlated with the default rate, but less with credit spreads and it is mildly procyclical.

Table 1: **Correlation of Raw Data**

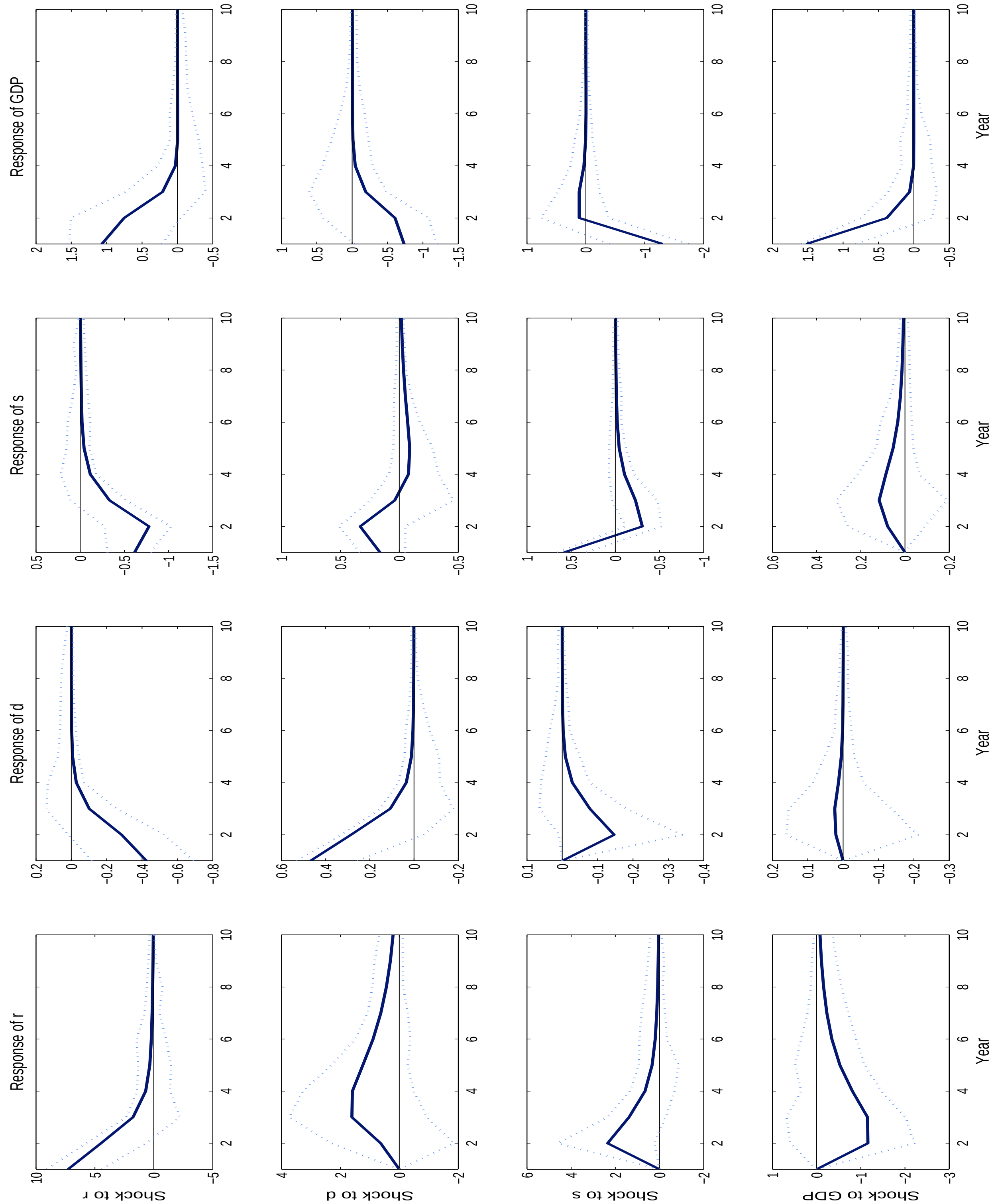
	Recovery Rate	Default Rate	Spread	GDP Growth
Recovery Rate	1	-0.73	-0.38	0.34
Default Rate	-	1	0.63	-0.60
Spread	-	-	1	-0.61
GDP Growth	-	-	-	1

In order to further understand their relationships, we order the four variables according to $[recovery, \quad default, \quad spread, \quad GDP]'$ and estimate a structural VAR model by using a simple Cholesky decomposition. We rank financial market variables before the macro variable. Different orderings of financial variables do not significantly change the results below. The following figure presents impulse responses functions from the VAR estimation exercise.

An immediate observation is that a shock that raises default rates also raises credit spreads. At the same time, it pushes up the recovery rate and depresses real GDP growth. In the model that we consider below, the default rate is most strongly affected by self-fulfilling prophecies. Believes into higher default rates leads to a cut in lending and higher credit spreads. The fall in credit induces a decline in economic activity. Since lending is reduced, given the constant recovery ability, lenders can recover more per unit of lending, which explains why the measured recovery rate goes up.

[3] We consider annual averages of the monthly series, updated until 2015 (see Simon Gilchrist's website http://people.bu.edu/sgilchri/Data/data.htm).

Figure 1: **VAR Evidence**. Note: “r” stands for “recovery rate”, “d” stands for “default rate”, “s” stands for “spreads”, and “GDP” stands for GDP growth rate. All variables are in percentage terms. The doted lines are 5% and 95% intervals



The above exercise shows that the recovery rate, credit spreads, and the default probability seem to have a non-trivial relationship, depending on the economy situation. This finding motivates us to develop a macroeconomic model with borrowing constrained firms and endogenous default, which takes into account different mechanisms by which default, recovery rates and spreads are affected by macroeconomic shocks. Credit market conditions impact firms with different productivity levels so that the real economy responds to changes in the credit market.

3 An Illustrative Example

We present a simple partial equilibrium model to illustrate how default rates, credit spreads and leverage can vary in response to changes in self-fulfilling expectations. The model has a large number of firms who live through infinitely many discrete periods $t \geq 0$. Firm owners are risk-averse and maximize discounted expected utility

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t [(1-\beta) \log c_t - \eta_t] \tag{1}$$

where c_t is consumption (dividend payout) in period t, $\beta < 1$ is the discount factor, and η_t is a default loss that materializes only when the firm defaults in period t. For example, the default loss may reflect the additional labor effort of the firm owner in a default event.[4] The default loss is idiosyncratic and stochastic: with probability p it is zero, otherwise it is $\Delta > 0$. Hence in any given period, fraction p of the firms are more susceptible to default.

All firms are endowed with one unit of net worth in period zero and they have access to a linear technology that transforms one unit of a good in period t into Π units of the good in period $t+1$. Firms may obtain one-period credit from perfectly competitive and risk-neutral investors who have an outside investment opportunity at rate of return $\bar{R} < \Pi$. Although firms cannot commit to repay their debt, there is a record-keeping technology that makes it possible to exclude defaulting firms from all future credit. That is, if a firm decides to default, it is subject to the default utility loss (if any) in the default period and it may not borrow in all future periods.

Investors offer standard debt contracts that specify the interest rate R and the volume of debt b. Competition between investors ensures that the offered contracts (R, b) maximize the borrower's utility subject to the investors' participation constraint. The latter requires

[4]Alternatively, we may assume in this example, as well as in the full macro model of the next section, that a defaulting firm's net worth is subject to a real default cost shock. This alternative model has the same credit market equilibrium but slightly different aggregate dynamics. Details are available upon request.

that the expected return equals the outside return $\bar{R}$ per unit of debt. In recursive notation, a firm owner's utility $V(\omega)$ depends on the firm's net worth ω and satisfies the Bellman equation

$$V(\omega) = \max_{s,(R,b)} (1-\beta)\log(\omega - s) + \beta \mathbb{E} \max \Big\{ V[\Pi(s+b) - Rb], V^d[\Pi(s+b)] - \eta \Big\} \ , \tag{2}$$

subject to $\mathbb{E}(R \cdot b) = \bar{R} \cdot b$. The expectation operator is over the firm's realization of the default loss $\eta \in \{0, \Delta\}$, and $V^d(.)$ is the utility value of a firm with a default history, which satisfies the recursion

$$V^d(\omega) = \max_s (1-\beta)\log(\omega - s) + \beta V^d(\Pi s) \ . \tag{3}$$

In Bellman equation (2), the second maximization expresses the optimal default choice at the beginning of the next period after the default cost η materializes: either the firm repays debt $-Rb$ and retains access to credit, or the firm defaults in which case it incurs the default loss η and is shut out of credit markets. Maximization in both (2) and (3) is over the firm's savings s, trading off utility from current consumption $\omega - s$ against the continuation value of next period's net worth. In (2), firms with access to credit also maximize over credit contracts (R, b) subject to the investors' participation constraint.

We show in the Appendix (proof of Proposition 2) that all firms save $s = \beta\omega$ and that value functions take the simple forms $V(\omega) = \log(\omega) + V$, $V^d(\omega) = \log(\omega) + V^d$, where V and V^d are independent of the firm's net worth. We write $v \equiv V - V^d$ to express the surplus value of access to credit; it is a forward-looking variable that reflects *expected credit conditions*. Using this notation, we can write the value function as $V(\omega) = \max_s (1-\beta)\log(\omega - s) + \beta[V^d + U(s)]$ where $U(s)$ is the surplus value of the optimal credit contract for a firm with savings s. It solves the problem

$$U(s) \equiv \max_{(R,b)} \mathbb{E} \max \Big\{ \log[\Pi(s+b) - Rb] + v, \log[\Pi(s+b)] - \eta \Big\} \quad \text{s.t.}$$

$$\bar{R}b = \mathbb{E}(Rb) = \begin{cases} Rb & \text{if } \log[\Pi(s+b) - Rb] + v \geq \log[\Pi(s+b)] \ , \\ (1-p)Rb & \text{if } \log[\Pi(s+b)] > \log[\Pi(s+b) - Rb] + v \geq \log[\Pi(s+b)] - \Delta \ , \\ 0 & \text{else.} \end{cases}$$

The participation constraint captures three possible outcomes. In the first case, the firm repays for any realization of the default loss so that investors are fully repaid Rb. In the second case, the firm only repays when the default loss is positive, which is reflected in the expected return $(1-p)Rb$. In the third case, the firm defaults with certainty.

It is straightforward to characterize the optimal contract.

Proposition 1. *Suppose that the parameter condition*

$$\frac{(e^{\Delta}-1)(1-p)}{e^{\Delta}-1+p} < \frac{\bar{R}}{\Pi} < \frac{(e^{(1-p)\Delta}-e^{-p\Delta})(1-p)}{e^{(1-p)\Delta}-1} \tag{4}$$

holds. Then there exists a threshold value $\bar{v} \in (0, v^{\max})$ with $v^{\max} \equiv \log(\Pi/(\Pi-\bar{R}))$, such that

(i) *If $v \in [\bar{v}, v^{\max})$, the optimal contract is $(R,b) = (\bar{R}, b(s))$ with debt level and borrower utility*

$$b(s) = s\frac{\Pi(1-e^{-v})}{\bar{R}-\Pi(1-e^{-v})}\ ,\ U(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R}-\Pi(1-e^{-v})}\right] .$$

(ii) *If $v \in [0, \bar{v})$, the optimal contract is $(R,b) = (\bar{R}/(1-p), b(s))$, with debt level and borrower utility*

$$b(s) = s\frac{\Pi(1-p)(1-e^{-v-\Delta})}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})}\ ,\ U(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta .$$

If expected credit conditions are good enough, $v \geq \bar{v}$, the threat of credit market exclusion is so severe that no firm defaults in the optimal contract. The corresponding debt level is the largest one that prevents default of firms with zero default loss whose binding enforcement constraint is $\log[\Pi(s+b) - Rb] + v = \log[\Pi(s+b)]$. A feasible solution to the optimal contracting problem further requires that debt is finite which necessitates $v < v^{\max}$.

Alternatively, if expected credit conditions are not so good, $v < \bar{v}$, the optimal contract allows for partial default since it is now relatively costly to prevent default of all firms. Instead, fraction p of firms default in the optimal contract, whereas firms with positive default cost are willing to repay which is ensured by $\log[\Pi(s+b)-Rb]+v = \log[\Pi(s+b)]-\Delta$.

The parameter conditions (4) imply that both outcomes are optimal for different expected credit conditions. If one of these inequalities fails, either no default (i) or partial default (ii) is the optimal contract for all feasible values of v.

Expected credit conditions v depend themselves on the state of the credit market and are determined in a stationary equilibrium by the forward-looking Bellman equations (2) and (3). After substitution of $U(s)$ from Proposition 1, it is immediate that the value difference $v = V - V^d$ satisfies the fixed-point equation

$$v = f(v) \equiv \begin{cases} \beta \log\left[\frac{\bar{R}}{\bar{R}-\Pi(1-e^{-v})}\right] & \text{if } v \geq \bar{v}\ , \\ \beta\left\{\log\left[\frac{\bar{R}}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta\right\} & \text{if } v < \bar{v}\ . \end{cases}$$

Any solution of this equation constitutes a stationary equilibrium of this economy. Under the conditions of Proposition 2, it is straightforward to verify that f is increasing and continuous, and it satisfies $f(0) > 0$ and $f(v) \to \infty$ for $v \to v^{\max}$. This shows that, generically, the fixed-point equation has either no solution, or two solutions. Moreover, if $f(\bar{v}) < \bar{v}$ holds, there is one equilibrium at $v^D < \bar{v}$ which involves default and a positive interest spread together with another equilibrium at $v^N > \bar{v}$ which has no default and a zero spread (see Figure 2). This result is summarized as follows.[5]

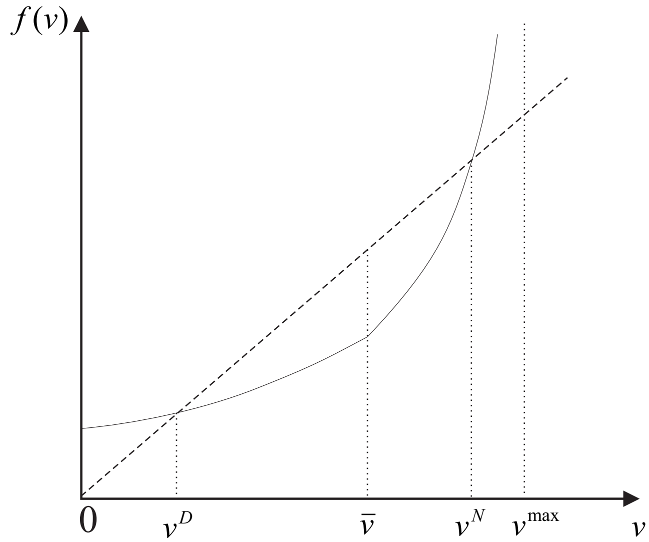


Figure 2: Co-existence of default and no-default equilibria.

Proposition 2. *Suppose that parameters satisfy*

$$\left(\frac{\bar{R}}{\bar{R} - \Pi(1 - e^{-\bar{v}})}\right)^{\beta} < \frac{\Pi[1 - (1-p)e^{-p\Delta}]}{\Pi - \bar{R} + e^{(1-p)\Delta}(\bar{R} - \Pi(1-p))} \ , \tag{5}$$

as well as condition (4). Then there are two stationary credit market equilibria $v^D < v^N$ such that default rates and interest spreads are positive at v^D and zero at v^N.

The main insight of this proposition is that the state of the credit market is a matter of self-fulfilling expectations. A well-functioning credit market with a low interest rate and a low default rate is highly valuable for firms, and this high valuation makes credit contracts without default self-enforcing. Conversely, a weak credit market with a higher interest rate and more default is valued less by the firms, and therefore it cannot sustain credit contracts that prevent default.

[5]If the parameter condition (5) (which is equivalent to $f(\bar{v}) < \bar{v}$) fails, there can exist at most two equilibria with default, or at most two equilibria without default. Since function f is convex and kinks upwards at $\bar{v}$, there cannot be more than two equilibria.

Although the two equilibria are clearly ranked in terms of default rates, interest rates and utility, it is worth noticing that leverage, defined as the debt-to-equity ratio $b(s)/s$, can be higher or lower in the no-default state compared to the default state. On one hand, the lower interest rate and the higher credit market valuation at the no-default equilibrium permit a greater leverage. On the other hand, preventing default of all firms requires a tighter borrowing constraint compared to the one that induces only firms with high default costs to repay.[6]

The additional parameter condition (5) of Proposition 2 is fulfilled whenever the discount factor β is low enough (because the fraction on the right-hand side is strictly greater than one). Conversely, the condition fails if β is sufficiently large.[7] In other words, a prerequisite for weak credit markets is that future consumption is discounted sufficiently strongly.

While this analysis describes stationary equilibria, we wish to remark that this partial equilibrium model also allows for self-fulfilling sunspot cycles in which the economy fluctuates perpetually between states of positive spreads and default and states with zero spreads and no default. We allow for such sunspot shocks in the general-equilibrium model that we describe next.

4 The Macroeconomic Model

We extend the insights of the previous section to a dynamic general equilibrium model. The main departures from the partial model are as follows: (i) the safe interest rate is endogenously determined by credit demand and supply; (ii) lenders can recover some of their exposure in a default event; (iii) defaulters are not permanently excluded; (iv) there are idiosyncratic productivity shocks so that the credit market impacts aggregate factor productivity; (v) there are different aggregate shocks that permit us to study business-cycle implications. These include shocks to fundamentals (technology and financial variables) as well as sunspot shocks.

[6] As a numeric example, set $\beta = 0.9$, $\Pi = 1$, $\bar{R} = 0.92$, $p = 0.1$, and two values of the default loss, $\Delta = 0.2$ and $\Delta = 0.4$. For both values of Δ, there is a no-default equilibrium at $v^N \approx 0.43$ with leverage $b/s \approx 0.61$. For $\Delta = 0.2$, the default equilibrium at $v^D \approx 0.11$ has lower leverage $b/s \approx 0.35$. For $\Delta = 0.4$, leverage at the default equilibrium $v^D \approx 0.2$ is $b/s \approx 0.79$. Hence, the default equilibrium can have *higher* leverage than the no-default equilibrium: the greater default loss relaxes the borrowing constraint which is imposed to preclude default of high-cost firms, while permitting default of the other firms.

[7] In fact, in this limiting case infinite debt levels would become sustainable, so that this partial model has no equilibrium at the given (low) interest rate $\bar{R} < \Pi$. In the general-equilibrium model of the next section there always exists an equilibrium since the endogenous interest rate would rise when β becomes sufficiently large.

4.1 The Setup

Firms and Workers

The model has a unit mass of infinitely-lived firm owners with the same preferences as in the previous section: period utility is $(1-\beta)\log(c)-\eta$ where c is consumption and β is the discount factor. The idiosyncratic default loss η is distributed with cumulative function G which is assumed to have no mass points.

All firms operate a production technology which produces output (consumption and investment goods) $y=(zk)^\alpha(A_t\ell)^{1-\alpha}$ from inputs capital k and labor ℓ with capital share $\alpha\in(0,1)$. A_t is time-varying aggregate productivity that grows over time and is hit by exogenous TFP shocks:[8]

$$\ln A_t=\mu_A+\ln A_{t-1}+\epsilon_t^A\ .$$

Firms can have high or low capital productivity z, and the idiosyncratic productivity state follows an i.i.d process. Specifically, a firm obtains high productivity z^H with probability π and low productivity $z^L=\gamma z^H$ with $1-\pi$. To simplify algebra, we assume that the capital productivity shock affects the stock of capital (rather than the capital service), so that the firm's capital stock at the end of the period is $(1-\delta)zk$, where δ is the depreciation rate.

Next to firm owners, the economy includes a mass of workers who supply a unit stock of labor $\bar{L}=1$ inelastically and who consume their labor earnings. That workers are hand-to-mouth consumers is not a strong restriction but follows from imposing a zero borrowing constraint on workers: If workers have the same discount factor β as firm owners, they do not wish to save in the steady-state equilibrium if the gross interest rate satisfies $\bar{R}<1/\beta$ so that workers' consumption equals labor income in all periods.[9]

Consider a firm operating the capital stock k. In the labor market, the firm hires workers at competitive wage rate w_t. This leads to labor demand which is proportional to the firm's effective capital input zk, so that the firm's net worth (before interest expense) is $\Pi_t zk$, where the gross return per efficiency unit of capital is (see the Appendix for details)

$$\Pi_t=\left[\alpha\left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}+1-\delta\right]\ . \tag{6}$$

[8] To simplify notation, we use time index t to indicate time-varying aggregate variables. Idiosyncratic variables carry no index since we formulate them in recursive notation below.

[9] This standard argument extends to a stochastic equilibrium around a steady-state equilibrium as long as shocks are not too large.

Credit Market

The credit market channels funds from low-productivity firms (lenders) to high-productivity firms (borrowers). Competitive banks pool the savings of lenders, taking the safe lending rate $\bar{R}_t$ as given, and offer credit contracts to borrowers. Issuing credit is costly: per unit of debt, the bank needs to pay intermediation cost Φ_t. For one, Φ_t captures administrative credit costs, such as the screening and monitoring of borrowers. Furthermore, although banks insure lenders against idiosyncratic default risk, they need to buy insurance against the aggregate component of default risk which can be obtained from unmodeled foreign insurance companies.[10] Therefore, Φ_t also includes such insurance costs, in addition to the administrative credit costs. Φ_t may be subject to shocks which stand for disturbances in financial intermediation. These shocks directly impact the interest spread between borrowing and lending rates.

Credit contracts take the form (R, b), where R is the gross borrowing rate, which reflects the firm's default risk, and b is the firm's debt. As in the previous section, the debt level in the optimal contract is proportional to the firm's internal funds (equity). Moreover, because all borrowing firms face the same ex-ante default incentives, the debt-to-equity ratio for all borrowing firms is the same and only depends on the aggregate state. This implies that we can write the equilibrium contract as (R_t, θ_t) where θ_t is the debt-to-equity ratio. We derive this optimal contract below.

If a firm borrows in period t and decides to default in period $t+1$, creditors can recover fraction λ_t of the borrower's gross return $\Pi_t zk$. The recovery rate λ_t stands for the fraction of collateral assets that can be seized in the event of a default. It may be subject to "financial shocks" which can be understood as disturbances to the collateral value or to the cost of liquidation.[11] The owner of the defaulting firm keeps share $(1-\lambda_t)\zeta$ of the assets, where $\zeta < 1$ is a real default cost. In subsequent periods, the firm carries a default flag which prevents access to credit. In any period following default, however, the default flag disappears with probability ψ in which case the firm regains full access to the credit market.[12]

[10] Without this assumption, which follows Jeske, Krueger, and Mitman (2013), banks cannot offer a safe lending rate to depositors in combination with standard credit contracts. In Appendix C, we describe the credit market in the absence of a foreign insurance market. In this alternative world, competitive banks offer risky securities to lenders (low-productivity firms) to fund credit to high-productivity firms.

[11] See e.g. Gertler and Karadi (2011) and Jermann and Quadrini (2012) for a similar modeling approach. See Chen (2010) for cyclical recovery rates.

[12] Such default events can stand for a liquidation (such as Chapter 7 of the U.S. Bankruptcy Code) of the firm in which case the owner may start a new business with harmed access to credit, or for a reorganization (such as Chapter 11) in which case the firm continues operation.

Timing

Within each period, the timing is as follows. First, the aggregate state $X_t = (A_t, \Phi_t, \lambda_t, \sigma_t)$ realizes. The first three components are the fundamental parameters described above which follow a joint Markov process. σ_t is a sunspot state which is uncorrelated over time. Next to the aggregate state vector, idiosyncratic default costs η realize and indebted firms either repay their debt or opt for default. Firms with a default history lose the default flag with probability ψ. Second, firms learn their idiosyncratic productivity $z \in \{z^L, z^H\}$ and make savings and borrowing decisions. Third, workers are hired and production takes place.

4.2 Equilibrium Characterization

Credit Market

Write $V(\omega; X_t)$ for the value of a firm with a clean credit record and net worth ω in period t after default decisions have been made. Similarly, $V^d(\omega; X_t)$ denotes the value of a firm with a default flag. A borrowing firm with net worth ω in period t chooses savings s and a credit contract (θ, R) to maximize

$$(1-\beta)\log(\omega-s)+\beta\mathbb{E}_t \max\left\{V\Big(z_t^H\Pi_t(1+\theta)-\theta R)s; X_{t+1}\Big), V^d\Big(z_t^H\Pi_t(1+\theta)(1-\lambda_t)\zeta s; X_{t+1}\Big)-\eta'\right\},$$

where the expectation is over the realization of the aggregate state X_{t+1} and the idiosyncratic default cost η' in period $t+1$. A borrower who does not default earns the leveraged return $z_t^H\Pi_t(1+\theta) - \theta R$ and has continuation utility $V(.)$, whereas a defaulter earns $z_t^H\Pi_t(1+\theta)(1-\lambda_t)\zeta$, incurs the default loss η' and has continuation utility $V^d(.)$.

We show in the Appendix that these value functions take the form $V^{(d)}(\omega, X_t) = \log(\omega) + V^{(d)}(X_t)$, and we write $v_t \equiv V(X_t) - V^d(X_t)$ to denote the surplus value of a clean credit record ("*credit market expectations*"). Write $\rho = R/(z_t^H\Pi_t)$ for the interest rate relative to the borrowers' capital return. Then the objective of a borrowing firm can be rewritten[13]

$$(1-\beta)\log(\omega-s)+\beta\log(s)+\beta\mathbb{E}_t \max\Big\{\log[1+\theta(1-\rho)], \log[(1+\theta)(1-\lambda_t)\zeta]-\eta'-v_{t+1}\Big\}.$$

It is immediate that every borrower saves $s = \beta\omega$. Moreover, there is an ex-post default threshold level

$$\tilde{\eta}' = \log\left[\frac{(1+\theta)(1-\lambda_t)\zeta}{1+\theta(1-\rho)}\right] - v_{t+1}\ , \tag{7}$$

such that the borrower defaults if and only if $\eta' < \tilde{\eta}'$. The threshold $\tilde{\eta}'$ varies with next

[13]The constant terms $\log(z_t^H\Pi_t) + \mathbb{E}V(X_{t+1})$ are irrelevant for the maximization and hence cancel out.

period's credit market value v_{t+1} and with the contract (θ, ρ).

Competitive banks offer contracts (θ, ρ). If a bank issues aggregate credit $B = \theta S$ (to borrowers with aggregate equity S), it needs to raise funds θS from lenders. In the next period $t+1$, the bank repays $\bar{R}_t \theta S$ to lenders, the intermediation cost $\Phi_t \bar{R}_t \theta S$ and it earns risky revenue $(1 - G(\tilde{\eta}'))R\theta S + G(\tilde{\eta}')\lambda_t(1+\theta)S$ where $\tilde{\eta}'$ is the ex-post default threshold for contract (θ, R). Since the bank is insured against aggregate default risk, competition drives expected bank profits to zero, which implies that

$$\bar{\rho}_t(1 + \Phi_t) = \mathbb{E}_t \Big\{ (1 - G(\tilde{\eta}'))\rho + G(\tilde{\eta}')\lambda_t \frac{1+\theta}{\theta} \Big\} \ , \tag{8}$$

where $\bar{\rho}_t \equiv \bar{R}_t/(z_t^H \Pi_t)$ measures the safe interest rate relative to the borrowers' capital return. The right-hand side of (8) is the expected gross revenue per unit of debt (relative to $z_t^H \Pi_t$). In default events $\eta' < \tilde{\eta}'$, banks can recover $\lambda_t(1+\theta)/\theta$ per unit of debt. The left-hand side are the costs of repaying lenders in period $t+1$ who fund credit and the intermediation cost in period t.

Banks offer contracts to maximize profits subject to the participation constraint of borrowers, and competition between banks drives profits to zero. Equivalently, the offered contracts maximize borrower utility,

$$\max_{(\theta,\rho),(\tilde{\eta}')} \mathbb{E}_t \Big\{ (1 - G(\tilde{\eta}')) \log[1 + \theta(1-\rho)] + \int_{-\infty}^{\tilde{\eta}'} \log[(1+\theta)(1-\lambda_t)\zeta] - \eta' - v_{t+1} \, dG(\eta') \Big\} \ ,$$

subject to the ex-post default choice (7) and the zero-profit condition for banks (8).

In the Appendix, we describe the optimal contract as follows:

Proposition 3. *Given a safe interest rate $\bar{\rho}_t$, intermediation cost Φ_t, collateral parameter λ_t, and (stochastic) credit market expectations v_{t+1}, the optimal credit contract in period t, denoted (θ_t, ρ_t), together with the ex-post (stochastic) default threshold $\tilde{\eta}_{t+1}$ satisfy the following equations:*

$$\tilde{\eta}_{t+1} = \log \Big[\frac{(1-\lambda_t)\zeta}{1-\xi_t} \Big] - v_{t+1} \ , \tag{9}$$

$$\theta_t = \frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t \left[\lambda_t G(\tilde{\eta}_{t+1}) + \xi_t(1 - G(\tilde{\eta}_{t+1})) \right]} - 1 \ , \tag{10}$$

$$\mathbb{E}_t[G'(\tilde{\eta}_{t+1})(\xi_t - \lambda_t)] = \mathbb{E}_t(1 - G(\tilde{\eta}_{t+1})) \Big\{ 1 - \bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t[G(\tilde{\eta}_{t+1})(\xi_t - \lambda_t)] \Big\} \ , \tag{11}$$

with $\xi_t \equiv \rho_t \theta_t/(1+\theta_t)$.

Conditions (9) and (10) are the ex-post default choice and the zero-profit condition of banks, respectively. Condition (11) is the first-order condition of the contract value maximization problem.[14]

As in the partial model of the previous section, credit market expectations v_t depend themselves on the state of the credit market, satisfying the recursive equation (see the Appendix for a derivation):

$$v_t = \beta\pi\mathbb{E}_t\Big\{ \log(1+\theta_t) + \log(1-\lambda_t) + \log\zeta - \tilde{\eta}_{t+1}(1-G(\tilde{\eta}_{t+1})) - \int_{-\infty}^{\tilde{\eta}_{t+1}} \eta \, dG(\eta)\Big\} + \beta(1-\psi-\pi)\mathbb{E}_t v_{t+1} \ . \tag{12}$$

The value of access to the credit market in period t includes two terms. First, with probability π the firm becomes a borrower in which case it benefits from higher leverage θ_t, whereas higher expected default thresholds $\tilde{\eta}_{t+1}$ reduce the value of borrowing. Second, the term $\beta(1-\psi-\pi)\mathbb{E}_t v_{t+1}$ captures the discounted value of credit market access from period $t+1$ onward.

General Equilibrium

In the competitive equilibrium, firms behave optimally as specified above, and the capital and labor market are in equilibrium.

Consider first the capital market. The gross lending rate $\bar{R}_t$ cannot fall below the capital return of unproductive firms $z_t^L \Pi_t$, which implies that $\bar{\rho}_t \geq \gamma = z^L/z^H$. When $\bar{\rho}_t > \gamma$, unproductive firms invest all their savings in the capital market; they only invest in their own inferior technology if $\bar{\rho}_t = \gamma$. Therefore, capital market equilibrium implies the following complementary slackness condition:

$$\gamma \leq \bar{\rho}_t, \quad s_t \pi \theta_t \leq (1-\pi) \ , \tag{13}$$

where $s_t \in [0,1]$ is the share of aggregate capital owned by firms with a clean credit record. The left-hand side of the second inequality is total borrowing plus intermediation costs (relative to total capital): fraction $s_t\pi$ of capital is owned by borrowers and θ_t is borrowing per unit of equity. The right-hand side $(1-\pi)$ is the share of capital owned by unproductive firms, which is fully invested in the capital market if the safe interest rate $\bar{\rho}_t$ exceeds γ. Otherwise, if $\underline{\rho}_t = \gamma$, a fraction of the capital of unproductive firms is invested in their own businesses.

[14] In our parameterizations with normally distributed default costs we verify that the second-order condition is also satisfied and that the solution is indeed a global maximum. Substituting (9) into (11) yields an equation in ξ_t whose solution implies θ_t via (10), and hence the interest rate $\rho_t = \xi_t(1+\theta_t)/\theta_t$.

Now, let K_t^H be the aggregate capital stock operated by productive firms and let Ω_t be the domestic aggregate net worth. Then, the capital stock of productive firms is $K_t^H = \beta\Omega_t\pi\Big[s_t(1+\theta_t)+1-s_t\Big]$. Savings of productive firms in period t are $\beta\Omega_t\pi$. Fraction s_t of this is owned by borrowing firms whose capital is $1+\theta_t$ per unit of internal funds. Fraction $1-s_t$ is owned by firms without access to credit whose capital is all internally funded. The capital stock operated by unproductive firms is $K_t^L = \beta\Omega_t\left[(1-\pi)-\pi s_t\theta_t\right]$. That is, these firms use the fraction of savings that is not invested in the capital market.

Since the labor market is frictionless, labor demand of any firm is proportional to the efficiency units of capital: $\ell = zk[(1-\alpha)A_t^{1-\alpha}/w_t]^{1/\alpha}$. Since labor supply is normalized to one, if we impose credit market clearing condition (13), the real wage that clears the labor market satisfies

$$w_t = (1-\alpha)A_t^{1-\alpha}\left(\beta\Omega_t\right)^{\alpha}\left(z^L\Big[(1-\pi)-\pi s_t\theta_t\Big]+z^H\pi\Big[s_t(1+\theta_t)+1-s_t\Big]\right)^{\alpha}. \tag{14}$$

It remains to describe the evolution of the aggregate domestic net worth Ω_t and the share s_t of net worth owned by firms with credit market access. The aggregate net worth in period $t+1$ is

$$\Omega_{t+1} = \beta z^H\Pi_t\Omega_t\left\{(1-\pi)\bar{\rho}_t+\pi s_t\Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t))+G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\zeta\Big]+\pi(1-s_t)\right\}. \tag{15}$$

In period t, all firms save fraction β of their net worth Ω_t. Fraction $1-\pi$ are unproductive and earn return $z^H\Pi_t\bar{\rho}_t = \bar{R}$. Fraction πs_t of aggregate savings is invested by borrowing productive firms of which fraction $1-G(\tilde{\eta}_{t+1})$ do not default and $G(\tilde{\eta}_{t+1})$ default in $t+1$. Fraction $\pi(1-s_t)$ of aggregate savings is invested by productive firms without credit market access who earn return $z^H\Pi_t$.

The net worth of firms with credit market access in period $t+1$ is

$$s_{t+1}\Omega_{t+1} = \beta z^H\Pi_t\Omega_t\left\{(1-\pi)s_t\bar{\rho}_t+\pi s_t(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t))+(1-s_t)\psi[(1-\pi)\bar{\rho}_t+\pi]\right\}.$$

The right-hand side of this equation is explained as follows. Fraction s_t of net worth is owned by firms with access to the credit market in period t. Fraction $1-\pi$ of these firms earn $\bar{\rho}_t z^H\Pi_t$, and fraction $\pi(1-G(\tilde{\eta}_{t+1}))$ of firms borrow and do not default, earning return $[1+\theta_t(1-\rho_t)]z^H\Pi_t$. All these firms retain access to the credit market in the next period. Fraction $1-s_t$ of net worth is owned by firms without access to credit in period t. They earn $\bar{\rho}_t z^H\Pi_t$ with probability $1-\pi$, and $z^H\Pi_t$ with probability π, and they regain access to the credit market with probability ψ. Adding up the net worth of all these firms gives the net worth of firms with credit market access in period $t+1$, $s_{t+1}\Omega_{t+1}$. Division of this expression by (15) yields

$$s_{t+1} = \frac{s_t\Big[(1-\pi)\bar{\rho}_t + \pi(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t))\Big] + (1-s_t)\psi[(1-\pi)\bar{\rho}_t + \pi]}{(1-\pi)\bar{\rho}_t + \pi s_t\Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\zeta\Big] + \pi(1-s_t)}. \tag{16}$$

A *competitive equilibrium* describes wages, credit contracts, aggregate net worth and capital, policy and value functions of firms such that: (i) firms make optimal savings and borrowing decisions, and borrowing firms decide optimally about default; (ii) banks make zero expected profits by offering standard debt contracts to borrowers and save interest rates to lenders; (iii) the labor and the capital market are in equilibrium. The characterization of equilibrium described above is summarized as follows.

Definition 1. *Given an initial state* (s_0, Ω_0) *and an exogenous stochastic process for the state vector* $X_t = (A_t, \Phi_t, \lambda_t, \sigma_t)$*, a competitive equilibrium is a stochastic process for (*$\tilde{\eta}_t$*,* θ_t*,* ρ_t*,* $\bar{\rho}_t$*,* v_t*,* Π_t*,* w_t*,* Ω_{t+1}*,* s_{t+1}*) as a function of (*s_t*,* Ω_t*), satisfying the equations (6), (9), (10), (11), (12), (13), (14), (15), (16).*

In Appendix B, we describe the steady-state solutions of this model, where we focus on steady states with $\bar{\rho} = \gamma$ which implies that some capital is used in less productive firms so that aggregate factor productivity responds endogenously to the state of the credit market. As in the illustrative example of the previous section, this more general model typically generates two steady state, one of which is locally indeterminate and hence susceptible to sunspot shocks.

5 Quantitative Analysis

In this section, we explore quantitative implication of the model. We first calibrate the (indeterminate) deterministic steady state to suitable long-run targets. Then we estimate financial shocks and sunspot shocks to account for the dynamics of default, recovery rates and credit spreads, in order to analyze the dynamics around the indeterminate steady state of the model.

5.1 Calibration Strategy

We assume that η is normally distributed with mean μ and variance σ^2. Given that we consider annual time series for default rates and recovery rates, we calibrate the model at annual frequency. There are 15 model parameters:

1. Preferences: β, μ, σ, κ, ν.
2. Technology: α, δ, μ^A, z^H, γ, π, ζ.
3. Financial markets: ψ, λ, Φ.

Directly calibrated are $\alpha = 0.3$ (capital share), $\delta = 0.1$ (annual depreciation rate), $\psi = 0.1$ (10-year exclusion), $\mu^A = 0.0211$ (growth rate of GDP per capita), $1 - \zeta = 0.15$ (direct net-worth losses in default, Davydenko, Strebulaev, and Zhao (2012)). We normalize average capital productivity at $\tilde{z} = \pi z^H + (1-\pi)z^L + s\pi\theta z^H(1-\gamma) = 1$. Notice that if we do not normalize $\tilde{z}$ in this way, then δ is not the depreciation rate of this economy.[15] The normalization implies

$$z^H = \frac{1}{\pi + (1-\pi)\gamma + s\pi\theta(1-\gamma)} \ . \tag{17}$$

The parameters for calculating z^H are discussed in the following steps.

We choose a reasonable target for the annual capital output ratio $K/Y = 2$. Then, $\Pi = 1 - \delta + \alpha\frac{Y}{\tilde{z}K} = 1 - \delta + \alpha\frac{Y}{K} = 1.07$. We target the debt-to-output ratio $B/Y = 0.82$, based on all firm credit 1981-2012, as well as the leverage ratio in constrained firms $\theta = 2.85$ (as in Azariadis, Kaas, and Wen (2016)). We set probability $\pi = 0.15$ so that only few firms are financing constrained (see Almeida, Campello, and Weisbach (2004)).

The recovery rate is measured by the price traded after the issuer defaults per dollar unit of bonds. We target the default rate $G = G(\tilde{\eta}) = 0.0171$ and the recovery rate $r = 0.42$. Because of $r = \lambda/\xi = \lambda(1+\theta)/(\theta\rho)$, we have

$$\gamma(1+\Phi) = (1-G)\rho + G\lambda(1+\theta)/\theta = (1-G)\rho + Gr\rho \ ,$$

which implies the interest spread (note $\bar{\rho} = \gamma$):

$$\Delta = \frac{\rho}{\gamma} = \frac{1+\Phi}{1-G+Gr} \ .$$

Given that the spread in the data is $\Delta = 1.019956$, we calibrate $\Phi = 0.0098$.

These four financial targets (debt-output ratio, leverage, default rate, recovery rate) identify the parameters μ, σ, λ and γ (or alternatively π), as we show now.

The steady-state value of s (share of firms with credit market access) follows from $\pi\theta s = \frac{Debt}{K} = \frac{Debt}{Y} \cdot \frac{Y}{K} = 0.41$, hence $s = 0.41/(\pi\theta) = 0.9720$. From the steady-state equation for s,

[15] $(1-\delta)\tilde{z}K_t$ of the capital survives to the next period. Hence, depreciation is $K_t - (1-\delta)\tilde{z}K_t$, and the depreciation rate is $1 - (1-\delta)\tilde{z}$ which equals δ iff $\tilde{z} = 1$.

we have the quadratic equation

$$as^2 + bs + c = 0$$

where $a = \pi\theta(1-\rho) + \pi G[\theta\rho - (1+\theta)\left[1 - \zeta(1-\lambda)\right]] > 0$, $b = \pi - \pi(1-G)\left[1+\theta(1-\rho)\right] + \psi[(1-\pi)\bar{\rho} + \pi]$, and $c = -\psi[(1-\pi)\bar{\rho} + \pi] < 0$. Use (17), $\rho = \Delta\gamma$, $\lambda = r\xi = \frac{r\theta\rho}{1+\theta}$, and the numbers for s, θ, ψ, π, to solve uniquely for

$$\gamma = \bar{\rho} = \frac{\left[\pi\theta - \pi G(1+\theta)(1-\zeta)\right]s^2 + \pi\left[1-(1-G)(1+\theta)\right]s - \psi\pi(1-s)}{\psi(1-\pi)(1-s) + \pi\theta\Delta(1-G)s(s-1) + \zeta\pi\theta G r\Delta s^2} = 0.5696$$

and therefore

$$\lambda = r\xi = \frac{r\theta\rho}{1+\theta} = \frac{r\theta\gamma(1+\Phi)}{(1+\theta)(1-G+Gr)} = 0.1813\ .$$

From stationarity of Ω_t/A_t follows

$$e^{\mu_A} = \beta z^H \Pi\left\{(1-\pi)\gamma + \pi s\Big[(1-G)\left[1+\theta(1-\rho)\right] + \zeta G(1+\theta)(1-\lambda)\Big] + \pi(1-s)\right\}$$

and hence $\beta = 0.9769$ (i.e. β is identified from the K/Y ratio).

Now μ and σ are identified from $G(\tilde{\eta})$ and $G'(\tilde{\eta})$, where $\tilde{\eta}$ is the steady-state default threshold. To see this, use the default threshold condition:

$$\ell \equiv \tilde{\eta} + v - \log\zeta = \log(1-\lambda) - \log(1-\xi) = 0.3622.$$

Then, the first-order condition for the optimal contract implies

$$G' = G'(\tilde{\eta}) = \frac{\left[1-\gamma(1+\Phi)\right](1-G)}{(1-\lambda)(1-e^{-\ell})} - G + G^2 = 1.6612.$$

The steady-state condition for v is

$$\frac{1+\beta\pi - \beta(1-\psi)}{\beta\pi}v = \log(1-\lambda) + \ln(1+\theta) + \log\zeta - \mu - (\tilde{\eta}-\mu)(1-G) + \sigma^2 G'\ .$$

Use $\tilde{\eta} = \ell - v + \log\zeta$ to solve this equation for v, which yields $\tilde{\eta}(\mu,\sigma) = \ell - v(\mu,\sigma)$. Then the numbers for G and G' yield $\mu = -0.6867$ and $\sigma = 0.0256$ (numeric solution).

Table 2: **Estimation Results**

	Prior Distribution	Prior Mean	Prior Std	Posterior Mean	5%	95%
ρ_λ	Normal	.70	.10	.6286	.5060	.7347
ρ_Φ	Normal	.70	.10	.6945	.5631	.8145
σ_λ	Inverse Gamma	.1000	.010	.1597	.1392	.1780
σ_Φ	Inverse Gamma	.0080	.002	.0070	.0061	.0079
σ_s	Inverse Gamma	.0500	.010	.0273	.0228	.0312

5.2 Estimation and Equilibrium Responses to Shocks

Now we explore the equilibrium dynamics in response to all three financial shocks. We estimate shocks to the recovery parameter λ_t, sunspots σ_t, and the intermediation cost Φ_t by using the time series data on the recovery rate, default rate, and spreads explained in Section 2. The first two of these shocks are essentially "credit demand" shocks, while the latter is a "credit supply" shock. We show impulse response functions when a one-time shock hits the economy and describe the transmission mechanism. Since the exogenous TFP parameter A_t has no impact on spreads, default rates or recovery rates, we do not consider these shocks here. Instead we ask how much of output dynamics can be accounted for by financial shocks alone.

We use Bayesian methods and estimate AR(1) processes for λ_t and Φ_t which satisfy

$$\ln \lambda_t - \ln \bar{\lambda} = \rho_\lambda \left(\ln \lambda_{t-1} - \ln \bar{\lambda} \right) + \varepsilon_t^\lambda \ ,$$

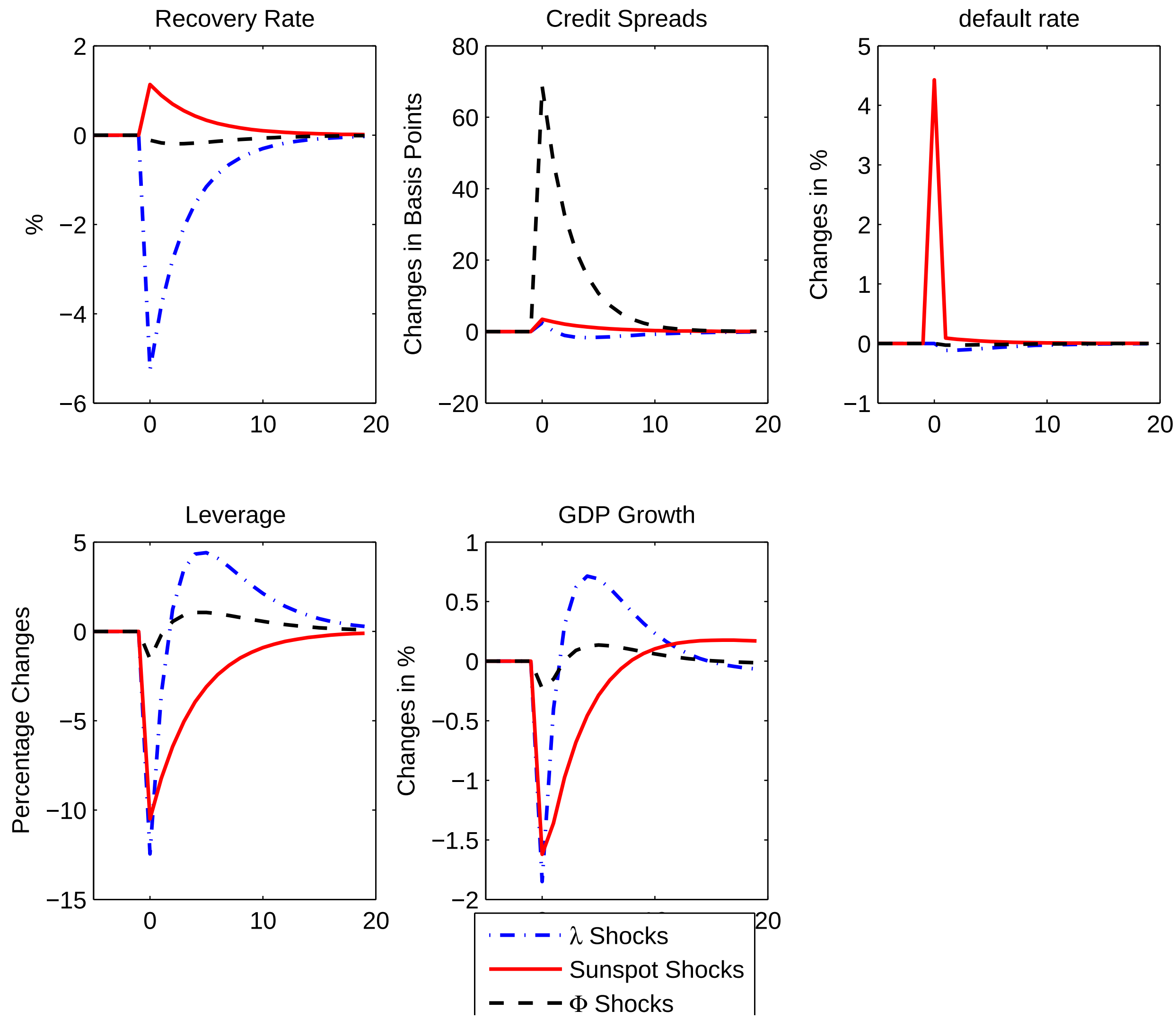
$$\ln(1 + \Phi_t) - \ln(1 + \bar{\Phi}) = \rho_\Phi \left[\ln(1 + \Phi_{t-1}) - \ln(1 + \bar{\Phi}) \right] + \varepsilon_t^\Phi \ ,$$

where ε_t^λ and ε_t^Φ are i.i.d. normally distributed with mean 0 and variances σ_λ^2 and σ_Φ^2. We define a positive sunspot shock as a shock that reduces the value of credit market access v_t and hence increases defaults. We denote the variance of the sunspot shock as σ_s^2.

Table 2 presents the estimation results. The posterior mean of σ_λ is updated to be larger than its prior means. The estimated standard deviations are not due to selection of priors. We have tried various different priors and the posterior estimates of σ_λ, σ_s and σ_Φ are robust. From these numbers we can infer that credit demand shocks are crucial to explain the data.

Using the posterior means, we plot the impulse response functions including recovery rate, credit spreads, default rate, leverage, and GDP growth to these three types of adverse financial shocks, i.e., a fall in λ, a fall in the credit market value (positive sunspot shock), and a rise in Φ. In all three cases, the economy starts from steady steady state and is hit by one standard deviation of only one type of shocks at time 0 (see Figure 3). We now illustrate

Figure 3: **Impulse Response Functions**



the transmission mechanism through these impulse response functions.

The one-time sunspot shock raises the default rate on impact by almost 4.5%, after which default falls back but remains persistently above the steady state. A remarkable outcome of sunspot shocks is that it persistently lowers the leverage ratio, because default incentives remain persistently high from time one onward. Lenders (who take these incentives into account) tighten the credit constraints and charge higher interest rates.[16] After the sunspot shock, the persistent response of all variables is the key to sustain self-fulfilling credit cycles. In fact, if the deterioration of credit conditions was rather short-lived, the value of credit market access does not fall much, which implies that default rates can go up only little. That

[16]Since the leverage ratio is much lower than the steady-state level, the recovery rate per unit of lending rises. Through tightening credit constraints, lenders are able to recover a greater share of their exposure in a default.

is, sizeable responses of default rates require persistent credit market cycles.

In the aggregate, productive firms as borrowers are hurt by this disturbance in the credit market. Because of the fall of leverage, these firms can use less capital. Therefore, we see a large endogenous fall in TFP which results in a significant 1.5% reduction in GDP growth. Notice that since there is no significant positive GDP growth later on, it takes a long time for the GDP level to return to its trend path.

As a comparison, the other two shocks generate much different outcomes. In response to a persistent fall in λ, credit spreads increase on impact but falls also quickly below the steady-state level. The changes compared to the steady-state level is almost negligible after the initial rise. Because of a lower λ, lenders tighten the leverage on impact and charge a higher interest rates to recovery the losses. But the fall in leverage only lasts for two years, as the autocorrelation of λ is rather small (0.6286). Lenders are thus willing to lend more soon after the shock. In expecting a higher leverage in the future, firms have less incentive to default, and this is why we have a persistent fall of default after negative λ shocks. The value of staying with clean credit records becomes higher after the fall in λ.

Finally, in response to a persistent rise in Φ, only the credit spread changes significantly, while all other variables barely move. This result is intuitive, since Φ is purely used to match the component of spreads that cannot be explained by default and recovery rates alone.

Overall, sunspot shocks are quantitatively important, since they can move leverage significantly and thus affect TFP and GDP growth to a large extent. The other two shocks, on the other hand, do not contribute to the volatility of GDP much. In addition, the correlations between default and the estimated λ shocks, the estimated sunspot shocks, and the estimated Φ shocks are 0.36, 0.99, and 0.37 , respectively. This implies that sunspot shocks are particularly important for default dynamics. Figure 4 shows the default rate and the estimated sunspot shocks. One can see that the default rate is indeed moved by sunspot shocks through the model.

Next we plot all three estimated shocks in Figure 5. Through the lens of our model, the 2007-2009 financial crisis is indeed special. It has a combination of a fall in recovery ability, rising beliefs of potential defaults, and larger-than-usual intermediation costs Φ. The Great Recession features the liquidity drop of financial assets (i.e., the fall in λ or the fall in pledgeability). Note also the increase of the estimated λ in the four years prior to the Great Recession which may reflect the surge of collateral assets in this period. The rise of intermediation costs Φ in 2008, which is induced by a sharp increase of the "excess bond premium", may reflect the banking crisis at the onset of the crisis.

Finally, we look at how much GDP growth rate data can be explained by the three financial shocks. Since we do not target GDP growth, there is no prior that our model could

Figure 4: **Sunspot and Default Rate**

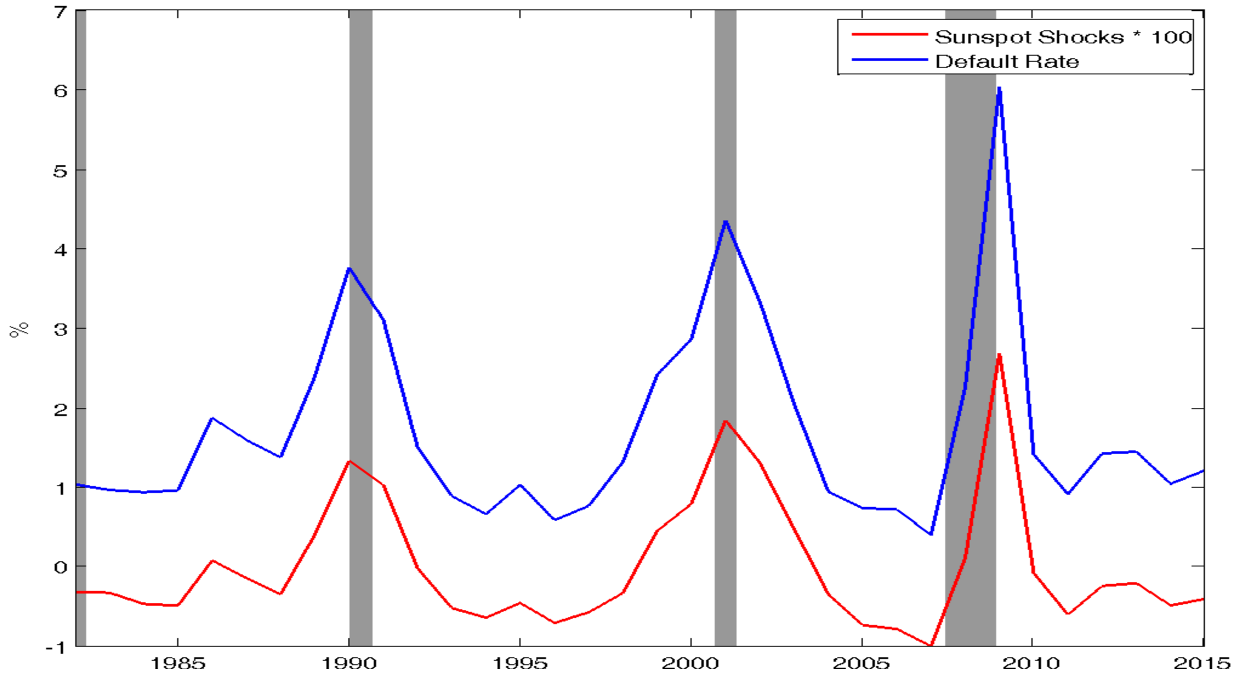


Figure 5: **Estimated Shocks**

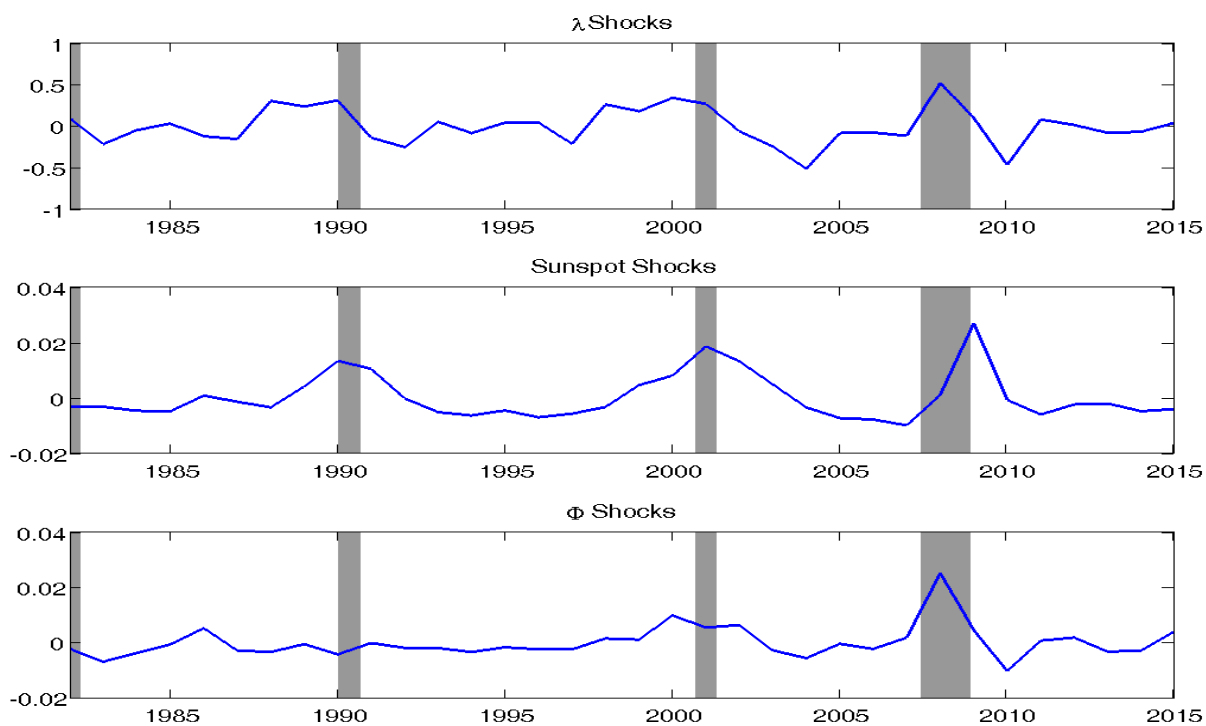
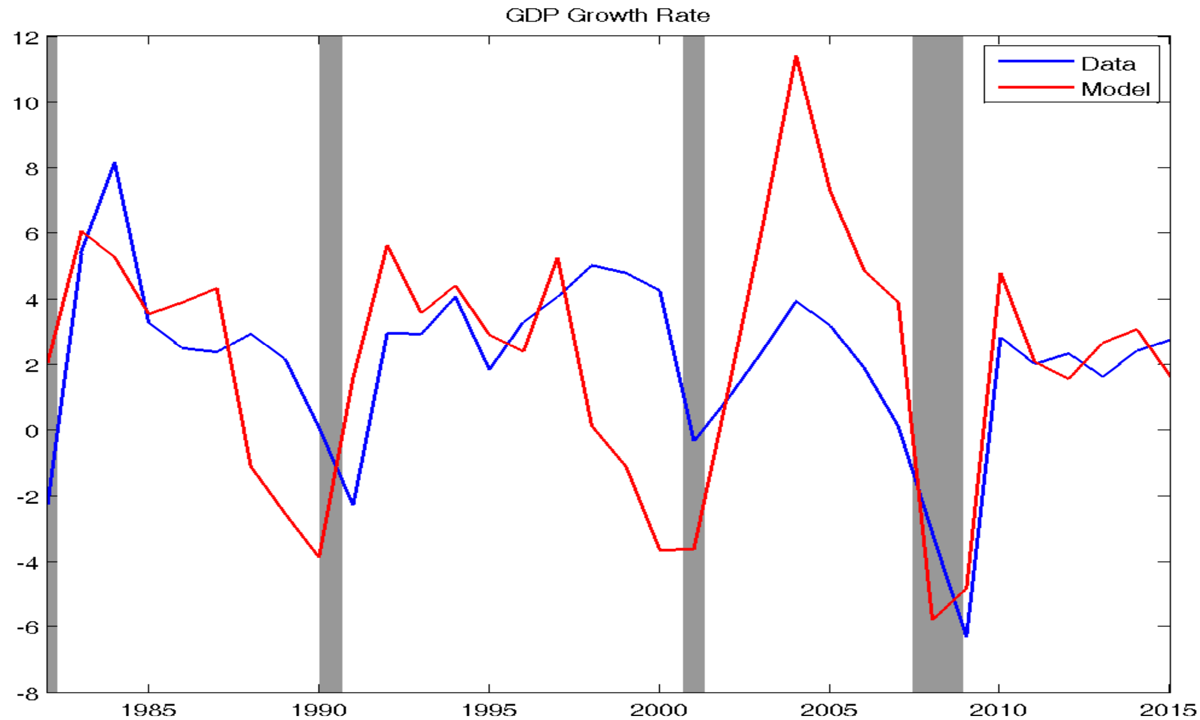


Figure 6: **GDP Growth**



generate reasonable GDP growth rate. Figure 6 shows that the three financial shocks in fact generate GDP dynamics that is quite close to the data. Because of exceptionally low default rates around 2005, the model predicts too much GDP growth. However, it captures particularly well the Great Recession period, as well as the business-cycle movements during the 1980s and 1990s. For variance decomposition, sunspot shocks can explain almost 56% of the variation in output growth while λ shocks explain about 42.7% of the variation. Intermediation cost shocks, however, can only explain 1.3% of the output growth variation.

When we add productivity shocks, i.e., $\ln A_t = \mu^A + \ln A_{t-1} + \varepsilon_t^A$, where ε_t^A is an i.i.d. mean zero normally distributed error term with variance σ_A^2, the model can fit the GDP series perfectly. Then, we find that the three financial shocks together can explain 83% of the total variation in GDP growth.

6 Conclusions

To be completed.

References

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2013): "Crisis and Commitment: Inflation Credibility and the Vulnerability to Sovereign Debt Crises," NBER Working Paper No. 19516.

Almeida, H., M. Campello, and M. S. Weisbach (2004): "The Cash Flow Sensitivity of Cash," *The Journal of Finance*, 59(4), 1777–1804.

Azariadis, C., L. Kaas, and Y. Wen (2016): "Self-Fulfilling Credit Cycles," *Review of Economic Studies*, 83(4), 1364–1405.

Benhabib, J., and P. Wang (2013): "Financial Constraints, Endogenous Markups, and Self-Fulfilling Equilibria," *Journal of Monetary Economics*, 60(7), 789–805.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, chap. 21, pp. 1341–1393. Elsevier.

Calvo, G. A. (1988): "Servicing the Public Debt: The Role of Expectations," *American Economic Review*, 78(4), 647–661.

Chen, H. (2010): "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure," *Journal of Finance*, 65(6), 2171–2212.

Christiano, L., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104, 27–65.

Cole, H. L., and T. J. Kehoe (2000): "Self-Fulfilling Debt Crises," *Review of Economic Studies*, 67(1), 91–116.

Conesa, J. C., and T. J. Kehoe (2015): "Gambling for Redemption and Self-Fulfilling Debt Crises," NBER Working Paper No. 21026.

Davydenko, S. A., I. A. Strebulaev, and X. Zhao (2012): "A Market-Based Study of the Cost of Default," *Review of Financial Studies*, 25(10), 2959–2999.

Duffie, D., A. Eckner, G. Horel, and L. Saita (2009): "Frailty Correlated Default," *Journal of Finance*, 64(5), 2089–2123.

Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann (2001): "Explaining the Rate Spread on Corporate Bonds," *Journal of Finance*, 56(1), 247–277.

Gertler, M., and P. Karadi (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58, 17–34.

Giesecke, K., F. A. Longstaff, S. Schaefer, and I. Strebulaev (2011): "Corporate Bond Default Risk: A 150-Year Perspective," *Journal of Financial Economics*, 102(2), 233–250.

Gilchrist, S., and E. Zakrajšek (2012): "Credit Spreads and Business Cycle Fluctuations," *The American Economic Review*, 102(4), 1692–1720.

Gomes, J. F., and L. Schmid (2012): "Equilibrium Credit Spreads and the Macroeconomy," Working Paper, Wharton School, University of Pennsylvania.

Gourio, F. (2013): "Credit Risk and Disaster Risk," *American Economic Journal: Macroeconomics*, 5(3), 1–34.

Gu, C., F. Mattesini, C. Monnet, and R. Wright (2013): "Endogenous Credit Cycles," *Journal of Political Economy*, 121(5), 940–965.

Harrison, S., and M. Weder (2013): "Sunspots and Credit Frictions," *Macroeconomic Dynamics*, 17(5), 1055–1069.

Huang, J.-Z., and M. Huang (2012): "How Much of the Corporate-Treasury Yield Spread is due to Credit Risk?," *Review of Asset Pricing Studies*, 2(2), 153–202.

Jermann, U., and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102(1), 238–71.

Jeske, K., D. Krueger, and K. Mitman (2013): "Housing, Mortgage Bailout Guarantees and the Macro Economy," *Journal of Monetary Economics*, 60(8), 917–935.

Khan, A., T. Senga, and J. K. Thomas (2016): "Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity," Working Paper, Ohio State University.

Liu, Z., and P. Wang (2014): "Credit Constraints and Self-Fulfilling Business Cycles," *American Economic Journal: Macroeconomics*, 6(1), 32–69.

Lorenzoni, G., and I. Werning (2013): "Slow Moving Debt Crises," NBER Working Paper No. 19228.

Miao, J., and P. Wang (2010): "Credit Risk and Business Cycles," Working Paper, Boston University.

Appendix A: Proofs and Derivations

Proof of Proposition 1: To characterize the optimal contract (R, b), note first that, conditional on an interest rate and on a default regime, the firm's utility is increasing in b. Hence, b should be as large as possible within a default regime, so that only one of the following three contracts can be optimal:

1. No credit: $b = 0$ with utility $U^0(s) = \log(\Pi s)$.
2. Partial default: $R = \bar{R}/(1-p)$, debt is at the largest level that prevents default in state $\eta = \Delta$, which is $b^D(s) = \frac{\Pi(1-p)(1-e^{-v-\Delta})}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})} \cdot s$. Utility is
$$U^D(s) = \log\left(\frac{\Pi s \bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\right) - (1-p)\Delta \ .$$
3. No default: $R = \bar{R}$, debt is at the largest level that prevents default for both states $\eta = 0, \Delta$, which is $b^N(s) = \frac{\Pi(1-e^{-v})}{\bar{R}-\Pi(1-e^{-v})} \cdot s$. Utility is
$$U^N(s) = \log\left(\frac{\Pi s \bar{R}}{\bar{R} - \Pi(1-e^{-v})}\right) \ .$$

Observe first that the level of savings s is irrelevant for the choice among these three contracts. Next, because of $U^N(s) \geq U^0(s)$ for all $v \geq 0$ (with strict inequality for $v > 0$), option 1 (no credit) can be ruled out (for any $v > 0$).

No default dominates partial default if $U^N(s) \geq U^D(s)$ which is equivalent to
$$v \geq \bar{v} = \log\left(\frac{\Pi e^{-\Delta}(p + e^{p\Delta} - 1)}{(\Pi - \bar{R})e^{-(1-p)\Delta} + \bar{R} - \Pi(1-p)}\right) \ .$$

$\bar{v}$ is well-defined because the expression in the $log(.)$ is positive: the denominator is positive if $(\Pi - \bar{R})e^{-(1-p)\Delta} > \Pi(1-p) - \bar{R}$. The latter condition follows from the first inequality in (4). Moreover, the first inequality in (4) is equivalent to $\bar{v} < v^{\max} = \log(\Pi/(\Pi - \bar{R}))$. Hence, no default is the optimal contract for all $v \in [\bar{v}, v^{\max})$.

The second inequality in condition (4) is equivalent to $\bar{v} > 0$. Because $U^D(s) > U^N(s)$ is equivalent to $v < \bar{v}$, the partial default contract is optimal for all $v \in [0, \bar{v})$. □

Proof of Proposition 2: Substituting $V(\omega) = \log(\omega) + V$, $V^d(\omega) = \log(\omega) + V^d$, and

$U(s)$ from Proposition 1 into Bellman equations (2) and (3) yields

$$\begin{aligned} \log(\omega) + V &= \max_s (1-\beta)\log(\omega - s) + \beta[\log(\Pi s) + V^d] \\ &\quad + \beta \max\left\{ \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-e^{-v})}\right], \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta \right\} , \\ \log(\omega) + V^d &= \max_s (1-\beta)\log(\omega - s) + \beta[\log(\Pi s) + V^d] \ . \end{aligned}$$

This shows that the savings policy $s = \beta\omega$ is optimal for both types of firms and that the terms $\log(\omega)$ cancel out on both sides of these Bellman equations, leaving the constant terms V and V^d to be determined from

$$\begin{aligned} V &= (1-\beta)\log(1-\beta) + \beta[\log(\beta\Pi) + V^d] \\ &\quad + \beta \max\left\{ \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-e^{-v})}\right], \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta \right\} , \\ V^d &= (1-\beta)\log(1-\beta) + \beta[\log(\beta\Pi) + V^d] \ . \end{aligned}$$

Differentiate the second from the first equation yields the fixed-point equation $v = f(v)$ for the value difference $v = V - V^d$.

It is immediate from the definition of f and parameter condition (4) that f is well-defined for $v \in [0, v^{\max})$, that $f(v) \to \infty$ for $v \to v^{\max}$, and that f is increasing and continuous. Furthermore $f(0) > 0$ if and only if

$$\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-\Delta})} > e^{(1-p)\Delta} ,$$

which is equivalent to the second inequality in (4) (which is also equivalent to $\bar{v} > 0$). Then the claim of the proposition follows if $f(\bar{v}) < \bar{v}$ holds. This inequality is equivalent to the one stated in (5). □

Derivation of the capital return Π_t

For a firm with capital k, the first-order condition for hiring labor is

$$(1-\alpha)A\left(\frac{zk}{A_t \ell}\right)^{\alpha} = w_t \ .$$

Therefore, labor demand is

$$\ell = zk\left[\frac{(1-\alpha)\,A_t^{1-\alpha}}{w_t}\right]^{1/\alpha} ,$$

and net worth before interest expense (or interest income) is

$$\left[\alpha\left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}+1-\delta\right]zk\equiv\Pi_t zk\ .$$

Proof of Proposition 3: The contract (θ,ρ) together with state-specific default thresholds $(\tilde{\eta}')$ maximize

$$\mathbb{E}_t\left\{(1-G(\tilde{\eta}'))\log[1+\theta(1-\rho)]+\int_{-\infty}^{\tilde{\eta}'}\log[(1+\theta)(1-\lambda_t)\zeta]-\eta-v_{t+1}\ dG(\eta)\right\}\ ,$$

subject to (7) and (8). Substitution of $1+\theta(1-\rho)$ via (7) gives the objective function

$$\mathbb{E}_t\left\{\log\left((1+\theta)(1-\lambda_t)\zeta\right)-\tilde{\eta}'(1-G(\tilde{\eta}'))-\int_{-\infty}^{\tilde{\eta}'}\eta\ dG(\eta)-v_{t+1}\right\}\ .$$

The additive terms $\log((1-\lambda_t)\zeta)$ and $-\mathbb{E}_t v_{t+1}$ are irrelevant for the maximization. Solving (8) for $1+\theta$, using $\rho=\xi(1+\theta)/\theta$, gives

$$1+\theta(\tilde{\eta})=\frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t)-\Psi(\xi)}\ ,$$

with

$$\Psi(\xi)\equiv\mathbb{E}_t\Big\{\lambda_t G(\tilde{\eta}(\xi))+\xi(1-G(\tilde{\eta}(\xi)))\Big\}\ ,$$

and

$$\tilde{\eta}(\xi)=\log\left[\frac{(1-\lambda_t)\zeta}{1-\xi}\right]-v_{t+1}\ ,$$

which is the ex-post default threshold. Substitution into the objective function yields a maximization problem in ξ:

$$\max_{\xi}-\log(\bar{\rho}_t(1+\Phi_t)-\Psi(\xi))-\mathbb{E}_t\Big\{\tilde{\eta}(\xi)(1-G(\tilde{\eta}(\xi)))+\int_{-\infty}^{\tilde{\eta}(\xi)}\eta\ dG(\eta)\Big\}\ .$$

The first-order condition for this problem is $\frac{\Psi'(\xi)}{\bar{\rho}_t(1+\Phi_t)-\Psi(\xi)}=\frac{1}{1-\xi}\mathbb{E}_t(1-G(\tilde{\eta}(\xi)))$. Then, using the derivative $\tilde{\eta}'(\xi)=1/(1-\xi)$:

$$\Psi'(\xi)=\mathbb{E}_t(1-G(\tilde{\eta}(\xi)))+\frac{1}{1-\xi}\mathbb{E}_t[G'(\tilde{\eta}(\xi))(\lambda_t-\xi)]\ .$$

Substituting this into the first-order condition yields (11) in the proposition. Furthermore, the default threshold (9) follows directly from (7), and $\theta_t = \bar{\rho}_t(1+\Phi_t)/(\bar{\rho}_t(1+\Phi_t)-\Psi(\xi_t))-1$, which leads to (10). □

Derivation of equation (12)

Recall that $V(\omega; X_t)$ and $V^d(\omega; X_t)$ are values of firms with (without) a clean credit record whose net worth is ω in period t. Therefore

$$\begin{aligned} V(\omega; X_t) &= \pi \hat{V}_b(\omega; X_t) + (1-\pi)\hat{V}_l(\omega; X_t) \ , \\ V^d(\omega; X_t) &= \pi \hat{V}_b^d(\omega; X_t) + (1-\pi)\hat{V}_l^d(\omega; X_t) \ , \end{aligned}$$

where $\hat{V}_\tau^{(d)}(\omega; X_t)$, $\tau = b, l$, are values of borrowing and lending firms after realization of idiosyncratic capital productivities. These satisfy the Bellman equations

$$\begin{aligned} \hat{V}_b(\omega; X_t) =& \max_s (1-\beta)\log(\omega - s) \\ &+ \beta \mathbb{E}_t \max\left\{ V([1+\theta_t(1-\rho_t)]z^H \Pi_t s; X_{t+1}), V^d((1+\theta_t)(1-\lambda_t)\zeta z^H \Pi_t s; X_{t+1}) - \eta' \right\} \ , \\ \hat{V}_l(\omega; X_t) =& \max_s (1-\beta)\log(\omega - s) + \beta \mathbb{E}_t V(\bar{R}_t s; X_{t+1}) \ , \\ \hat{V}_b^d(\omega; X_t) =& \max_s (1-\beta)\log(\omega - s) + \beta(1-\psi)\mathbb{E}_t V^d(z^H \Pi_t s; X_{t+1}) + \beta\psi \mathbb{E}_t V(z^H \Pi_t s; X_{t+1}) \ , \\ \hat{V}_l^d(\omega; X_t) =& \max_s (1-\beta)\log(\omega - s) + \beta(1-\psi)\mathbb{E}_t V^d(\bar{R}_t s; X_{t+1}) + \beta\psi \mathbb{E}_t V(\bar{R}_t s; X_{t+1}) \ . \end{aligned}$$

Expectation operators are over the realizations of aggregate states and of the idiosyncratic default loss η' in period $t+1$.

It is straightforward to verify that all value functions take the form $\hat{V}_\tau^{(d)}(\omega; X_t) = \log(\omega) + \hat{V}_\tau^{(d)}(X_t)$ for $\tau = b, l$, $V^{(d)}(\omega'; X_t) = \log(\omega') + V^{(d)}(X_t)$, and that savings are $s = \beta\omega$. With $B \equiv (1-\beta)\log(1-\beta) + \beta\log(\beta)$, it follows

$$\hat{V}_b(X_t) = B + \beta \mathbb{E}_t \max\Big\{ \log([1+\theta_t(1-\rho_t)]z^H \Pi_t) + V(X_{t+1}), \tag{18}$$

$$\log((1+\theta_t)(1-\lambda_t)\zeta z^H \Pi_t) + V^d(X_{t+1}) - \eta' \Big\} \ , \tag{19}$$

$$\hat{V}_l(X_t) = B + \beta \log \bar{R}_t + \beta \mathbb{E}_t V(X_{t+1}) \ , \tag{20}$$

$$\hat{V}_b^d(X_t) = B + \beta \log(z^H \Pi_t) + \beta(1-\psi)\mathbb{E}_t V^d(X_{t+1}) + \beta\psi \mathbb{E}_t V(X_{t+1}) \ , \tag{21}$$

$$\hat{V}_l^d(X_t) = B + \beta \log \bar{R}_t + \beta(1-\psi)\mathbb{E}_t V^d(X_{t+1}) + \beta\psi \mathbb{E}_t V(X_{t+1}) \ . \tag{22}$$

Moreover,

$$V(X_t) = \pi \hat{V}_b(X_t) + (1-\pi)\hat{V}_l(X_t) \ , \tag{23}$$

$$V^d(X_t) = \pi \hat{V}_b^d(X_t) + (1-\pi)\hat{V}_l^d(X_t) \ . \tag{24}$$

Define $v_t = V(X_t) - V^d(X_t)$, take the difference between (23) and (24) and use (18)–(22) to obtain

$$\begin{aligned}
v_t =& \pi\Big\{\hat{V}_b(X_t) - \hat{V}_b^d(X_t)\Big\} + (1-\pi)\Big\{\hat{V}_l(X_t) - \hat{V}_l^d(X_t)\Big\} \\
=& \beta\pi\mathbb{E}_t\Bigg\{(1-\psi)v_{t+1} + \max\Big\{\log[1+\theta_t(1-\rho_t)], \log[(1+\theta_t)(1-\lambda_t)\zeta] - \eta' - v_{t+1}\Big\}\Bigg\} \\
& + \beta(1-\pi)(1-\psi)\mathbb{E}_t v_{t+1} \\
=& \beta\pi\mathbb{E}_t \max\Big\{\log[1+\theta_t(1-\rho_t)], \log[(1+\theta_t)(1-\lambda_t)\zeta] - \eta' - v_{t+1}\Big\} + \beta(1-\psi)\mathbb{E}_t v_{t+1} \ .
\end{aligned}$$

Using the default threshold $\tilde{\eta}_t$, the max{.} term is equal to

$$(1 - G(\tilde{\eta}_{t+1}))\log[1+\theta_t(1-\rho_t)] + \int_{-\infty}^{\tilde{\eta}_{t+1}} \log[(1+\theta_t)(1-\lambda_t)\zeta] - \eta - v_{t+1}\, dG(\eta) \ .$$

This proves equation (12).

Appendix B: Miscellaneous

Collection of Equilibrium Conditions

We list all equilibrium conditions used for numerical exercises. There are 10 equations and we have 10 unknowns ($\tilde{\eta}_t$, θ_t, ρ_t, $\bar{\rho}_t$, v_t, Π_t, w_t, Ω_{t+1}, s_{t+1}, ξ_t)

$$\tilde{\eta}_{t+1} = \log\left[\frac{1-\lambda_t}{1-\xi_t}\right] - v_{t+1} + \log\zeta \ ,$$

$$\theta_t = \frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t\left[\lambda_t G(\tilde{\eta}_{t+1}) + \xi_t(1-G(\tilde{\eta}_{t+1}))\right]} - 1$$

$$\mathbb{E}_t G'(\tilde{\eta}_{t+1})(\xi_t - \lambda_t)] = \mathbb{E}_t(1-G(\tilde{\eta}_{t+1}))\Big\{1 - \bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t G(\tilde{\eta}_{t+1})(\xi_t - \lambda_t)]\Big\}$$

$$v_t = \beta\pi\mathbb{E}_t\Big\{\log(1+\theta_t) + \log(1-\lambda_t) + \log\zeta - \tilde{\eta}_{t+1}(1-G(\tilde{\eta}_{t+1})) - \int_{-\infty}^{\tilde{\eta}_{t+1}} \eta\, dG(\eta)\Big\} + \beta(1-\psi-\pi)\mathbb{E}_t v_{t+1}$$

$$\xi_t = \rho_t\theta_t/(1+\theta_t)$$

$$\Pi_t = \left[\alpha\left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} + 1 - \delta\right]$$

$$\gamma \leq \bar{\rho}_t, \quad s_t\pi\theta_t \leq (1-\pi)$$

$$\frac{w_t}{A_t} = \left[\beta\frac{\Omega_t}{A_t}\Big(z^L\Big[(1-\pi) - \pi s_t\theta_t\Big] + z^H\pi\Big[s_t(1+\theta_t) + 1 - s_t\Big]\Big)\right]^{\alpha}(1-\alpha)$$

$$\left[\frac{\Pi_t - (1-\delta)}{\alpha}\right]^{\frac{\alpha}{\alpha-1}} = \beta\frac{\Omega_t}{A_t}\Big(z^L\Big[(1-\pi) - \pi s_t\theta_t\Big] + z^H\pi\Big[s_t(1+\theta_t) + 1 - s_t\Big]\Big)$$

$$\Omega_{t+1} = \beta z^H\Pi_t\Omega_t\left\{(1-\pi)\bar{\rho}_t + \pi s_t\Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + \zeta G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\Big] + \pi(1-s_t)\right\}$$

$$s_{t+1} = \frac{s_t\Big[(1-\pi)\bar{\rho}_t + \pi(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t))\Big] + (1-s_t)\psi[(1-\pi)\bar{\rho}_t + \pi]}{(1-\pi)\bar{\rho}_t + \pi s_t\Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + \zeta G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\Big] + \pi(1-s_t)}$$

Deterministic Steady State

In a deterministic steady state, the first 4 equations can be solved separately. To see this, one can first solve

$$\xi = \lambda + \frac{1-G}{G' + G(1-G)}\left[1 - \bar{\rho}(1+\Phi)\right]$$

$$\theta(\tilde{\eta}) = \frac{\bar{\rho}(1+\Phi)}{\bar{\rho}(1+\Phi) - \lambda - \frac{(1-G)^2}{G'+G(1-G)}\left[1-\bar{\rho}(1+\Phi)\right]} - 1$$

$$v = -\tilde{\eta} + \log(1-\lambda) - \log(1-\xi) + \log\zeta$$

Then, one solves $\tilde{\eta}$ from

$$\frac{1-\beta(1-\psi-\pi)}{\beta\pi}v(\tilde{\eta}) = \log(1+\theta(\tilde{\eta})) + \log\zeta + \log(1-\lambda) - \mu - (\tilde{\eta}-\mu)\left[1-G(\tilde{\eta})\right] + \sigma^2 G'(\tilde{\eta})$$

Output and Productivity

Given an equilibrium, we can describe the dynamics of aggregate variables. Aggregate output is simply

$$Y_t = (A_t\bar{L})^{1-\alpha}(\tilde{z}_t K_t)^{\alpha}\ , \tag{25}$$

with average capital productivity

$$\tilde{z}_t = \pi z^H + (1-\pi)z^L + s_t\pi\theta_t[z^H - z^L(1+\Phi_t]\ .$$

Define *total factor productivity* (TFP) as the residual of the aggregate production function, i.e.,

$$\tilde{A}_t = \frac{Y_t}{K_t^\alpha \bar{L}^{1-\alpha}} = A_t^{1-\alpha} \tilde{z}_t^\alpha \ .$$

Three things affect capital efficiency $\tilde{z}_t$ of this economy. First, a greater share of firms with access to the credit market leads to a more efficient capital allocation. Second, the higher the ability to raise external capital θ_t, the more capital is employed by productive firms. Third, lower intermediation costs Φ_t imply that less capital is eventually used at high-productivity firms.

Consumption and investment

Splitting output into consumption and investment is not straightforward. This is because firm owners consume out of their net worth at the beginning of the period, while workers are paid out of current production within the period. Conceptually consumption should be measured based on actual output, so this is why consumption in period t should be measured

$$C_t = (1-\alpha)Y_t + (1-\beta)\Omega_{t+1} \ ,$$

where Ω_{t+1} is net worth at the end of period t (beginning of period $t+1$). Gross investment in period t is

$$I_t = [K_{t+1} - K_t] + [1 - (1-\delta)\tilde{z}_t]K_t \ ,$$

where the first part is net investment which equals $\beta\Omega_{t+1} - K_t$ and the second part is depreciation. The latter takes into account that capital depreciates differently in high- or low-productivity firms since z shocks affect the stock of capital. Adding up consumption and investment, using gives $C_t + I_t = \Omega_{t+1} - (1-\delta)\tilde{z}_t K_t + (1-\alpha)Y_t$. We can write net worth in period $t+1$ as

$$\Omega_{t+1} = \Pi_t K_t \tilde{z}_t + Tr_{t+1} \ ,$$

where

$$Tr_{t+1} = \Pi_t K_t \pi s_t z^H \Big\{ \theta_t \bar{\rho}_t (1+\Phi_t) - (1 - G(\tilde{\eta}_{t+1}))\rho_t \theta_t - G(\tilde{\eta}_{t+1})\lambda_t (1+\theta_t) \Big\}$$

are net transfers from foreign insurance companies (i.e. payments from abroad to cover bank losses or payments of domestic banks to insurance firms if there are bank profits) which are identical to net imports. Because of $\Pi_t K_t \tilde{z}_t = \alpha Y_t + (1-\delta)\tilde{z}_t K_t$,

$$C_t + I_t = Y_t + Tr_{t+1} \ .$$

In words, consumption and investment equals domestic output plus net imports.

Appendix C: No insurance against aggregate risk

Here we describe a model variation in which domestic banks cannot buy insurance against aggregate risk from abroad. Instead, competitive banks in period t offer standard debt contracts (ρ, θ) to borrowers (as before), they pool the idiosyncratic default risk and sell mutual funds to investors who receive a state-contingent return $\bar{R}'$ in period $t+1$ which is subject to aggregate risk. We use the notation $\bar{R}' = \bar{\rho}' z^H \Pi_t$. Since banks cannot make losses, and competition drives profits to zero, they must repay all revenue to lenders in all states of the world. Instead of (8), this requires

$$\bar{\rho}'(1+\Phi_t) = (1 - G(\tilde{\eta}'))\rho + G(\tilde{\eta}')\lambda_t \frac{1+\theta}{\theta} \ . \tag{26}$$

Φ_t is again the proportional intermediation cost (determined in period t). The ex-post default level $\tilde{\eta}'$ depends on the contract (ρ, θ), and on the credit market value v_{t+1} according to (7) as before:

$$\tilde{\eta}' = \log\Big[\frac{(1+\theta)(1-\lambda_t)\zeta}{1+\theta(1-\rho)}\Big] - v_{t+1} \ . \tag{27}$$

Lenders (unproductive firms) can either invest in their own business which yields safe return $\gamma z^H \Pi_t$ or into mutual funds with risky return $\bar{\rho}' z^H \Pi_t$. If a lender with wealth ω invests fraction $\kappa \in [0,1]$ of savings s into his/her own business, expected utility is

$$(1-\beta)\log(\omega - s) + \beta \mathbb{E}_t \Big\{ \log([\kappa\gamma + (1-\kappa)\bar{\rho}']s) + V(X_{t+1}) \Big\} \ ,$$

(similar for lenders with a default flag). Obviously, lenders save $s = \beta\omega$, and they are choosing κ to maximize

$$\mathbb{E}_t \log\Big(\kappa\gamma + (1-\kappa)\bar{\rho}'\Big) \ ,$$

which gives rise to the first-order condition[17]

$$\mathbb{E}_t \Big\{ \frac{\gamma - \bar{\rho}'}{\kappa\gamma + (1-\kappa)\bar{\rho}'} \Big\} = 0 \ . \tag{28}$$

[17] This characterizes an interior solution $\kappa \in (0,1)$. Boundary solutions $\kappa = 0$ require a "less than or equal to" inequality in (28).

The optimal contract offered by competitive banks maximizes the utility of a borrower subject to the lenders' participation constraint.[18] Formally, the contract (θ, ρ) with state-contingent $\bar{\rho}'$ and default thresholds $\tilde{\eta}'$ solves

$$\max_{(\theta,\rho),(\tilde{\eta}',\bar{\rho}')} \mathbb{E}_t \Big\{ (1 - G(\tilde{\eta}')) \log[1 + \theta(1-\rho)] + \int_{-\infty}^{\tilde{\eta}'} \log[(1+\theta)(1-\lambda_t)\zeta] - \eta' - \upsilon_{t+1} \, dG(\eta') \Big\} \ ,$$

subject to the ex-post default threshold (27), the zero-profit condition for banks (26), and the participation constraint for lenders

$$\mathbb{E}_t \log \Big(\kappa\gamma + (1-\kappa)\bar{\rho}' \Big) = \bar{U}_t^l \ , \tag{29}$$

where $\kappa \in (0,1)$ satisfies (28).

We characterize the optimal contract as follows:

Proposition 4. *Given $\bar{U}_t^l$, intermediation cost Φ_t, collateral parameter λ_t, and (stochastic) credit market expectations υ_{t+1}, the optimal credit contract in period t, denoted (θ_t, ρ_t), together with the ex-post (stochastic) default threshold $\tilde{\eta}_{t+1}$, state-contingent rate of return $\bar{\rho}_{t+1}$ on mutual funds, and lenders portfolio decision κ_t satisfy the following equations:*

$$\tilde{\eta}_{t+1} = \log \Big[\frac{(1-\lambda_t)\zeta}{1-\xi_t} \Big] - \upsilon_{t+1} \ , \tag{30}$$

$$\bar{\rho}_{t+1}(1+\Phi_t) = \frac{1+\theta_t}{\theta_t} \Big\{ (1 - G(\tilde{\eta}_{t+1}))\xi_t + G(\tilde{\eta}_{t+1})\lambda_t \Big\} \ , \tag{31}$$

$$0 = \mathbb{E}_t \Big\{ \frac{\gamma - \bar{\rho}_{t+1}}{\kappa_t\gamma + (1-\kappa_t)\bar{\rho}_{t+1}} \Big\} \ , \tag{32}$$

$$(1+\Phi_t)\mathbb{E}_t \Big\{ \frac{\bar{\rho}_{t+1}}{\kappa_t\gamma + (1-\kappa_t)\bar{\rho}_{t+1}} \Big\} = \frac{1+\theta_t}{1 - \mathbb{E}_t G(\tilde{\eta}_{t+1})} \mathbb{E}_t \Big\{ \frac{(\lambda_t - \xi_t)G'(\tilde{\eta}_{t+1}) + (1 - G(\tilde{\eta}_{t+1}))(1-\xi_t)}{\kappa_t\gamma + (1-\kappa_t)\bar{\rho}_{t+1}} \Big\} \ , \tag{33}$$

with $\xi_t \equiv \rho_t\theta_t/(1+\theta_t)$.

Proof: Substitute (27) into the objective function of borrowers, replacing $\rho = \xi(1+\theta)/\theta$, and $\bar{\rho}'$ from (26) shows that the maximization problem can be written

$$\max_{\theta,\xi,\kappa} \log(1+\theta) + \mathbb{E}_t \Big\{ -\tilde{\eta}' + \int_{-\infty}^{\tilde{\eta}'} \tilde{\eta}' - \eta' dG(\eta') \Big\} \ ,$$

[18] Equivalently, the contract maximizes bank profits subject to borrowers' and lenders' participation constraints. Competition between banks drives profits to zero (in all states of the world) which gives rise to the equilibrium utility values of borrowers and lenders.

with $\tilde{\eta}' = -\log(1-\xi) + \log(1-\lambda_t) + \log\zeta - v_{t+1}$ and subject to

$$\mathbb{E}_t \log\left\{\kappa\gamma + \frac{(1-\kappa)(1+\theta)}{(1+\Phi_t)\theta}\Big[G(\tilde{\eta}')\lambda_t + (1-G(\tilde{\eta}'))\xi\Big]\right\} = \bar{U}_t^l \ .$$

Write μ for the multiplier on this constraint. Maximization with respect to κ directly yields (32). Maximization with respect to θ yields

$$\frac{1}{1+\theta} + \mu\mathbb{E}_t\left\{\frac{-(1-\kappa)[G(\tilde{\eta}')\lambda_t + (1-G(\tilde{\eta}'))\xi]\frac{1}{\theta^2(1+\Phi_t)}}{\kappa\gamma + (1-\kappa)\bar{\rho}'}\right\} = 0 \ . \tag{34}$$

For maximization with respect to ξ, note that $\frac{d\tilde{\eta}'}{d\xi} = 1/(1-\xi)$, and $\frac{d}{d\tilde{\eta}'}\mathbb{E}_t\{-\tilde{\eta}' + \int^{\tilde{\eta}'} \tilde{\eta}' - \eta' dG(\eta')\} = -1 + \mathbb{E}_t G(\tilde{\eta}')$. Using these expressions, the first-order condition with respect to ξ is

$$\frac{\mathbb{E}_t G(\tilde{\eta}') - 1}{1-\xi} + \mu\mathbb{E}_t\left\{\frac{(1-\kappa)\Big[(\lambda_t - \xi)G'(\tilde{\eta}')\frac{1}{1-\xi} + 1 - G(\tilde{\eta}')\Big]\frac{(1+\theta)}{\theta(1+\Phi_t)}}{\kappa\gamma + (1-\kappa)\bar{\rho}'}\right\} = 0 \ . \tag{35}$$

Combining (34) and (35) to eliminate μ gives

$$(1+\Phi_t)\mathbb{E}_t\left\{\frac{\bar{\rho}'}{\kappa\gamma + (1-\kappa)\bar{\rho}'}\right\} = \frac{1+\theta}{1-\mathbb{E}_t G(\tilde{\eta}')}\mathbb{E}_t\left\{\frac{(\lambda_t - \xi)G'(\tilde{\eta}') + (1-G(\tilde{\eta}'))(1-\xi)}{\kappa\gamma + (1-\kappa)\bar{\rho}'}\right\} \ .$$

This proves (33). □

The competitive equilibrium describes wages, credit contracts, returns on mutual funds, portfolio choices, aggregate net worth and capital, policy and value functions of firms such that: (i) firms make optimal savings and borrowing decisions, and borrowing firms decide optimally about default; (ii) banks make zero expected profits by offering standard debt contracts to borrowers and save interest rates to lenders; (iii) the labor and the capital market are in equilibrium. We describe an equilibrium where $\kappa_t \in (0,1)$, so that credit market equilibrium requires

$$s_t \pi \theta_t = (1-\kappa_t)(1-\pi) \ . \tag{36}$$

The dynamics of Ω_t and s_t changes to

$$\Omega_{t+1} = \beta z^H \Pi_t \Omega_t \Bigg\{ (1-\pi)[\kappa_t \gamma + (1-\kappa_t)\bar{\rho}_{t+1}] \tag{37}$$

$$+ \pi s_t \Big[(1 - G(\tilde{\eta}_{t+1}))(1 + \theta_t(1-\rho_t)) + G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\zeta \Big] + \pi(1-s_t) \Bigg\} \,,$$

$$s_{t+1} = \frac{s_t \Big[(1-\pi)[\kappa_t\gamma + (1-\kappa_t)\bar{\rho}_{t+1}] + \pi(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) \Big] + (1-s_t)\psi[(1-\pi)\bar{\rho}_t + \pi]}{(1-\pi)[\kappa_t\gamma + (1-\kappa_t)\bar{\rho}_{t+1}] + \pi s_t \Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_t)\zeta \Big] + \pi(1-s_t)} \,. \tag{38}$$

Definition 2. *Given an initial state* (s_0, Ω_0) *and an exogenous stochastic process for the state vector* $X_t = (A_t, \Phi_t, \lambda_t, \sigma_t)$*, a competitive equilibrium is a stochastic process for (*$\tilde{\eta}_t$*,* θ_t*,* ρ_t*,* $\bar{\rho}_t$*,* κ_t*,* υ_t*,* Π_t*,* w_t*,* Ω_{t+1}*,* s_{t+1}*) as a function of (*s_t*,* Ω_t*), satisfying the equations (6), (30), (31), (32), (33), (12), (36), (14), (37), (38).*